

# ADAPTATION OF MCFT-BASED SHEAR DESIGN EQUATION FOR FATIGUE LIFE PREDICTION OF RC BRIDGE DECK SLABS

Nuttapong KONGWANG\*<sup>1</sup> and Yasuhiko SATO\*<sup>2</sup>

## ABSTRACT

This study aims to assess the suitability of the MCFT-based shear design equation for predicting fatigue life of RC bridge deck slabs. Utilizing Takeda's previously established equations, which originated from an extensive experimental dataset of RC slabs subjected to moving wheel-type loading, this research adapts the MCFT-based shear design equation for fatigue life prediction. The results suggest that with modification to accommodate moisture and support conditions during fatigue loading, the MCFT-based shear design equation proves effective in predicting the fatigue life of RC bridge deck slabs.

**Keywords:** MCFT-based shear design equation, RC bridge deck slabs, fatigue life prediction

## 1. INTRODUCTION

Reinforced concrete (RC) bridge deck slabs commonly endure repeated concentrated vehicle wheel loads, resulting in fatigue damage [1,2] and ultimately causing punching shear failure in RC bridge deck slabs. The occurrence of fatigue failure in RC slabs has garnered significant attention from researchers, who have investigated the fatigue life of RC slabs under various parameters through both experimental and numerical analyses [3-7]. Consequently, several design guidelines, e.g., JSCE, fib model code 2010 and Eurocodes, include the requirement of shear fatigue strength verification in their provision. Meanwhile, AASHTO LRFD Bridge Design Specifications (AASHTO LRFD) [8] only provides the provision mainly related to steel reinforcement and steel structures. For the RC bridge deck slabs under the fatigue limit state conforming to AASHTO LRFD, they do not need to be investigated for the fatigue life because the laboratory testing with a series of fixed pulsating load fatigue test of model RC slabs proved that the RC decks slab had fatigue limits of approximately twice the service level [6]. However, transverse cracks were observed in the deck slabs of an RC box girder bridge in the US, after a few years of use, as documented in [9], this bridge also started experiencing multiple deck potholes. Therefore, the provision for fatigue life prediction is still essential when RC bridge deck slabs being designed following the AASHTO LRFD.

Sonoda and Horikawa [3] investigated differences in crack patterns resulting from fatigue loading tests using fixed and multipoint pulsating loads. Continuously, Matsui [4] developed a fatigue testing machine capable of simulating the effects of wheel loads. Using this device, Matsui effectively replicated punching shear

failures observed in reinforced concrete (RC) deck slabs. Additionally, the resulting crack pattern generated closely resembled the cracks observed in real RC bridge deck slabs. As a result, the development of fatigue life prediction of RC bridge deck slabs should be based on slabs subjected to moving wheel load more appropriately than that of under fixed pulsating loads.

As mentioned previously, although AASHTO does not explicitly include provisions for predicting fatigue life of RC bridge decks, it is commonly embraced as a benchmark design specification in numerous highway bridge projects. Thus, this study performs an adaptation of the AASHTO shear design provision which is the Modified Compression Field Theory (MCFT)-based shear design equation [10,11] for fatigue life prediction of RC bridge deck slabs through the past experimental data of RC slabs under moving wheel load type and existing fatigue life prediction method proposed by Takeda and Sato [12]. However, their method was developed from JSCE-based equation. Therefore, the objective of this study is also to modify their proposal by using MCFT-based shear design equation in order to provide the fatigue life prediction method of RC bridge deck slabs for the other countries where the AASHTO LRFD is being used.

## 2. FATIGUE LIFE PREDICTION OF RC SLABS UNDER MOVING WHEEL LOADING

### 2.1 Failure mechanism of RC slabs under moving wheel loading

Previous study [4] revealed that the typical cracking pattern of model RC slabs failed under moving wheel load in the experiment program is the grid-like cracking pattern, whereas the work done by Batchelor et al [6] that a radial-like pattern of cracks occurred in the

\*1 Graduate School of Creative Science and Engineering, Waseda University, JCI Student Member

\*2 Professor, Dept. of Civil and Environmental Engineering, Waseda University, Ph.D., JCI Member

fatigue failure under fixed pulsating load. In addition to the grid-like cracking pattern, previous researchers [4,5] have been clarified that the bottom cracks in the direction of the main reinforcement (transverse cracks) not only opened and closed cyclically but also vertically slide up and down during the wheel load repeatedly moved forward and backward on the deck slab. The deck's degradation was accelerated by that of the reverse shear mechanism in the transverse cracks where it compelled to propagate upwards, eventually forming a beam-formed (or segmental) cracking pattern as shown in Fig. 1. Finally, the damaged deck slab could no longer withstand the wheel loading, and punch-off failure occurred. This evident suggested that the fatigue life prediction method of RC slabs is appropriately based on linear members such as beams instead of the planar members. Thereby, the shear strength equation for fatigue life prediction of RC slabs under moving wheel loads using the MCFT-based shear design equation is proposed in this study, as explained later in the following section.

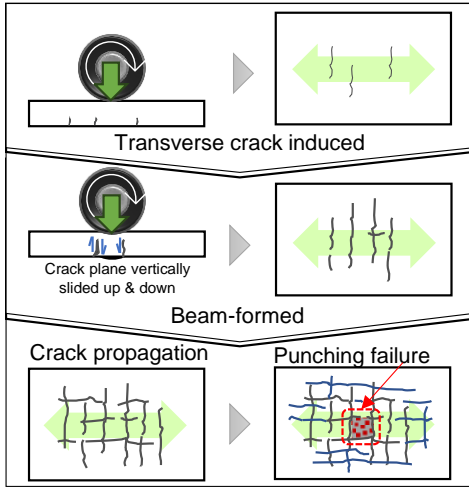


Fig.1 Typical fatigue failure mechanism and crack patterns under moving wheel loading

## 2.2 Existing fatigue life prediction method

Prediction methods for the fatigue life of RC deck slabs subjected to repeatedly moving wheel loads proposed by Takeda and Sato [12] including the shear strength equation,  $V_{bc}$ , (Eq. 1), coupling with the  $S-N$  equation (Eq. 5) for the beam-formed member in order to predict the fatigue life of RC slabs. Eq. 1 was developed based on the JSCE-based equation by combining the influence of environmental conditions and support conditions with the modification factor  $\alpha_e$  and  $\alpha_B$ , respectively. Furthermore, the influence of main and distributing reinforcement was included in the shear strength calculation by multiplying with the coefficient of  $\beta_{p1}$  and  $\beta_{p2}$ , which can be obtained from Eqs. 2 and 3, respectively.

$$V_{bc} = \alpha_e \alpha_B \beta_{p1} \beta_{p2} \beta_d f_{vmcd} b_{w,e} d \quad (1)$$

where,  $\alpha_e = 1.0$  in the dry condition and 0.69 in the wet condition,  $\alpha_B = 0.64, 1.00,$  and  $1.50$  for the slab supported by 2-side simple and 2-side free, 2-side simple

and 2-side elastic, and 4-side simple, respectively,  $\beta_d = \sqrt[4]{1000/d} \leq 1.5$ , the  $f_{vmcd} = 0.32 \sqrt[3]{f'_{cd}}$ , where  $f'_{cd}$  represents the compressive strength of concrete (N/mm<sup>2</sup>).

$$\beta_{p1} = 100 p_1 \left[ \frac{1}{3} + 0.5(100 p_2) \right] \quad (2)$$

$$\beta_{p2} = 1 + 0.125 \frac{p_2}{p_1} \quad (3)$$

$$B = b + 2d_d \quad (4)$$

where,  $d$  and  $d_d$  are the effective depth of main and distributing reinforcement (mm), respectively, and  $b$  denotes the loading plate length in the longitudinal direction (wheel-travel direction) (mm),  $p_1$  and  $p_2$  represents the main reinforcement ratio and the distributing reinforcement ratio, respectively. For the width of a beam-formed member,  $b_{w,e}$  (mm) can be computed by Eq. 4 proposed by Matsui [4].

$$S = P/2V_{bc} = 1 - K \log N \quad (5)$$

where,  $K = 0.057$  and  $0.061$  for the dry condition and the wet condition, respectively, and  $N$  denotes the number of cyclic moving loads at failure. The shear strength  $V_{bc}$  in Eq. 7 is multiplied by a constant of 2 to account for the fact that the constant load  $P$  was applied at the mid-span of the RC slabs in the tests.

Since Takeda and Sato [12] unveiled that the boundary and wet condition considerably affect the shear strength, Eq. 5 that describes the relationship between the experimentally obtained fatigue life  $N$  and the normalized shear amplitude  $S$  developed by the same authors was mainly adapted in this study. Nevertheless, their equations (Eq. 1-5) were developed by modifying the JSCE-based equation. As a consequence, in order to conduct the approach to predict the fatigue life of RC bridge deck slabs for using the MCFT-based shear design equation, the  $V_{bc}$  in Eq. 5 should be modified as described in the following section. Moreover, the equations proposed by Takeda and Sato [12] were employed for investigating the parameters affecting the shear strength for fatigue life prediction as well as the reliability and accuracy of the proposed equation.

## 3. EXTENDED MCFT-BASED SHEAR DESIGN EQUATION FOR FATIGUE LIFE PREDICTION

### 3.1 MCFT-based shear design equation

According to AASHTO LRFD [8], section 5.12.8.6, shear resistance,  $V_c$ , for a slab-type region without transverse reinforcement under the one-way action can be calculated by using Eq. 6. This equation is derived from the Modified Compression Field Theory (MCFT) of Vecchio and Collins [10], where the shear strength relies on the ability of diagonal cracked concrete to transmit tension and shear stresses taking place in the critical section. The expressions are as follows:

$$V_c = \beta \lambda \sqrt{f'_c} b_v d_v \quad (6)$$

where the shear resistance factor is denoted by  $\beta$ .  $\lambda$  denotes concrete density modification in which 1 can be used for normal density concrete, and  $f'_c$ ,  $b_v$ , and  $d_v$  represent the compressive strength of concrete (N/mm<sup>2</sup>),

effective width (mm), and effective shear depth (mm), respectively.

The value  $\beta$  in Eq. 6 has been developed to utilize a more convenient equation by Bentz et al. [11] in which a simplified expression (Eq. 7) for sections do not contain at least the minimum amount of shear reinforcement was given.

$$\beta = \frac{0.4}{1+750\varepsilon_s} \frac{1300}{1000+S_{xe}} \quad (7)$$

where  $\varepsilon_s$  is the net longitudinal tensile strain in the section at the centroid of the main reinforcement, which is calculated from  $(M_u/d_v+0.5N_u+|V_u|)/E_sA_s$ ;  $M_u$ ,  $N_u$  and  $V_u$  are the moment (N-mm), axial force (N), and shear force (N) at the section, respectively.  $E_s$  and  $A_s$  are the elastic modulus (N/mm<sup>2</sup>) and area of the main reinforcement (mm<sup>2</sup>), respectively.  $S_{xe}=S_x[35/(a_g+16)]$ , in which  $S_x$  denotes the crack spacing parameter taken as the greater of  $d_v$ ,  $0.9d$ , and  $0.72$  of member height (mm), and  $a_g$  is the maximum aggregate size (mm).

Regarding previous study [13], the comparative study of shear strength equations has highlighted the notable impact of support and moisture conditions on the shear strength of RC slabs, as depicted in Fig. 2. The utilization of MCFT-based shear strength equation (Eq. 6) yields the shear strengths represented by solid line in Fig. 2 is clearly different comparing to that of shear strengths (broken lines in Fig. 2) calculated by using the Eq. 1 which accounts for the effect of support and moist condition. It becomes evident that modifications are necessary to incorporate the effects of both support and moisture conditions into fatigue life prediction. Consequently, in order to account for these influences, a comprehensive dataset comprising numerous instances of RC slab data under varied support conditions, including those tested in wet conditions from previous experiments (details provided in the following section), was employed to refine Eq. 6 for its application in fatigue life prediction.

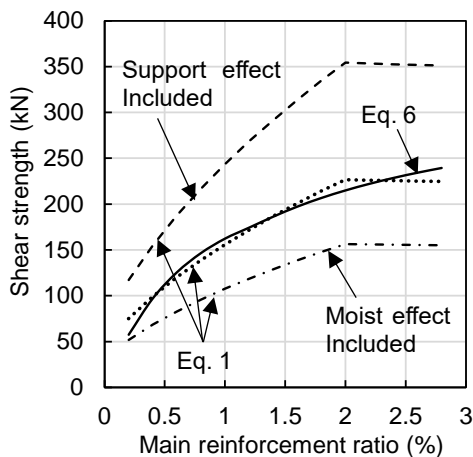


Fig.2 Influence of moisture condition and support condition [13]

### 3.2 Experimental dataset

The experimental data used in a previous study [12] was adopted in this study to investigate shear fatigue strength prediction using the MCFT-based shear design

equation. The experimental dataset, including 83 RC slabs that failed under a moving wheel loading tested with a constant load (70 slabs tested under dry condition and 13 slabs tested under wet condition), was collected and summarized in Table 1. The data are categorized by support condition into three main groups: 1) 2-side simple and 2-side free supports; 2) 2-side simple and 2-side elastic supports; and 3) 4-side simple supports, as graphically shown in Fig. 3. Additionally, Group 2 includes the slabs that were tested under wet conditions.

Table 1 Experimental data used

| Group  | 1         | 2         | 3         |
|--------|-----------|-----------|-----------|
| t      | 167-205   | 70-220    | 150       |
| $f'_c$ | 26.2-52.5 | 13.6-54.0 | 24.8-35.0 |
| $p_m$  | 0.84-1.74 | 0.74-1.59 | 0.85      |
| $p_d$  | 0.29-1.36 | 0.28-1.41 | 0.94      |
| P      | 157       | 24-323    | 100       |
| N      | 0.30-1449 | 0.01-3250 | 63.1-379  |

Note: t = the slab thickness (mm),  $f'_c$  = compressive strength of concrete,  $p_m$  = main reinforcement ratio (%),  $p_d$  = distribution reinforcement ratio (%), P = constant applied load (kN), and N = the number of cyclic moving loads at failure ( $10^3$  cycles).

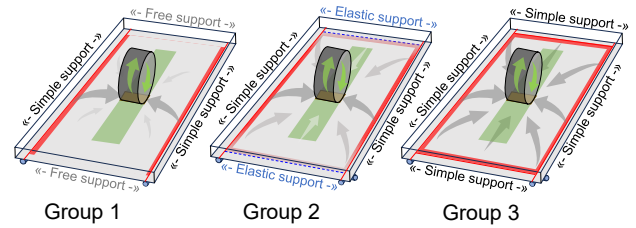


Fig.3 Support condition of the RC slab specimens in each group

### 3.3 Fatigue life prediction through MCFT-based shear design equation

#### (1) Proposed shear strength equation

As discussed earlier, the shear strengths estimated by original MCFT shear design equation (Eq. 6) are distinct from the existing equation. That contrast is attributed to the effect of support conditions and the wet conditions. Hence, the modified MCFT-based shear strength equation (Eq. 8),  $V_{mc}$  was proposed incorporating with the modification factors  $\alpha_{sc}$  and  $\alpha_{wc}$  to take into consideration the effect of support and moisture conditions respectively.

$$V_{mc} = \alpha_{wc} \alpha_{sc} \beta_{dc} \lambda \sqrt{f'_c} b_v d_v \quad (8)$$

where,  $\alpha_{wc} = 1.0$  in the dry condition and 0.59 in the wet condition,  $\alpha_{sc} = 0.50, 1.00,$  and  $1.30$  for the slab supported by 2-side simple and 2-side free, 2-side simple and 2-side elastic, and 4-side simple, respectively.

$$\beta_{dc} = 2.0 \frac{0.4}{1+(750-175 \frac{p_d}{p_m}) \varepsilon_s} \frac{1300}{1000+S_{xe}} \quad (9)$$

where  $\varepsilon_s$  and  $S_{xe}$  can be obtained in the same manner with Eq. 7.  $p_d$  and  $p_m$  denote the distribution reinforcement ratio main reinforcement ratio (%) respectively.

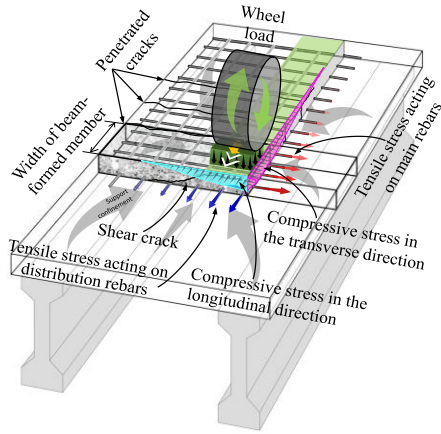


Fig.4 Shear bearing capacity mechanism of beam-formed member in the RC bridge deck slab under moving wheel load

Eq. 8 was developed based on the shear bearing capacity mechanism of the beam-formed member in the RC bridge deck slab under moving wheel load, as shown in Fig. 4. The development was started by evaluating the fatigue life of experimental data gained from the wheel load running test (Table 1) using the MCFT-based shear design equation (Eq. 6). Although the beam-formed member allows fatigue life prediction through the shear strength equation of the linear members, the fatigue life prediction results in great scattering in Fig. 5a when unmodified MCFT shear design equation,  $V_c$  was adapted in fatigue life calculation. Furthermore, in Fig. 5a, only the Group 1 slabs show good agreement with the S-N curve (Eq. 5) because those slabs are supported by 2-side simple and 2-side free supports, which similarly behave as the typical simply supported beams. These variances could be due to the difference in support conditions resulting in different support confinement. This confinement emerges as the arching force enhancing the load-bearing capacity of the laterally restrained RC slab [14,15]. The arching force generates the compressive struts that occur in the slab plane between the loading point and surrounding supports, as shown in Fig. 4. Those struts can directly transfer some applied load to the support, reducing strain in the main reinforcement. Consequently, coefficient 2.0 is introduced in the modified strain factor,  $\beta_{dc}$  (Eq. 9) to

deal with the confinement owing to the support condition. Fig. 5b displays the fatigue life prediction yielded from replacing the parameter  $\beta$  in Eq.6 ( $V_c$ ) with the  $\beta_{dc}$ . Although the results show that all groups move closer to the S-N curve, they still have some deviation in slabs groups 1 and 3, as it can be seen from the ratio of experimental and calculated normalized shear forces ( $S_{test}/S_{cal}$ ) in Table 2. They diverge from 1.00 with the values of 0.50 and 1.30 for groups 1 and 3 respectively. This divergence is caused by the difference in support, which affects the aspect and level of confinement, as shown in Fig. 3. It has also been confirmed by previous studies [16,17]. Finally, those deviated values become the modification factors,  $\alpha_{sc}$ , accounting for the effect of support conditions.

Additionally, there are two main differences between typical RC beams and RC beam-formed in cracked RC deck slabs. First, the distribution reinforcement is placed perpendicular to the main reinforcement as shown in Fig.4. Second, they are connected to the surrounding slab, and each member is always in close contact with adjacent ones. As a results, the distribution reinforcement can still carry and distribute the applied load to the beam-formed member nearby resulted in the decrease in strain of the main reinforcement,  $\epsilon_s$ ; this strain is one of the main parameters of the MCFT-based shear design equation. Therefore, it is necessary to modify the coefficient of  $\epsilon_s$  in  $\beta$ . Eventually, the coefficient of  $\epsilon_s$  in  $\beta_{dc}$  is reduced depending on the ratio between distribution and main reinforcement ( $\rho_d/\rho_m$ ) multiplied by 175 which denotes the load distribution ability of the distribution reinforcement. It was gained from the regression analysis of the experimental data (Table1) used in this study.

Table 2 Average  $S_{test}/S_{cal}$  using different shear strength equations related to Fig. 5

| Average $S_{test}/S_{cal}$ | $S_{test}=P/2V_c$<br>Fig. 5a | $S_{test}=P/2V_{c,\beta_{dc}}$<br>Fig. 5b | $S_{test}=P/2V_{mc}$<br>Fig. 5c |
|----------------------------|------------------------------|---|---------------------------------|
| Group 1                    | 0.99                         | 0.50                                      | 1.00                            |
| Group 2                    |                              |   |                                 |
| - Dry                      | 2.22                         | 1.00                                      | 1.00                            |
| - Wet                      | 1.28                         | 0.59                                      | 1.00                            |
| Group 3                    | 3.19                         | 1.30                                      | 0.99                            |

Note:  $S_{cal}$  calculated using Eq. 5

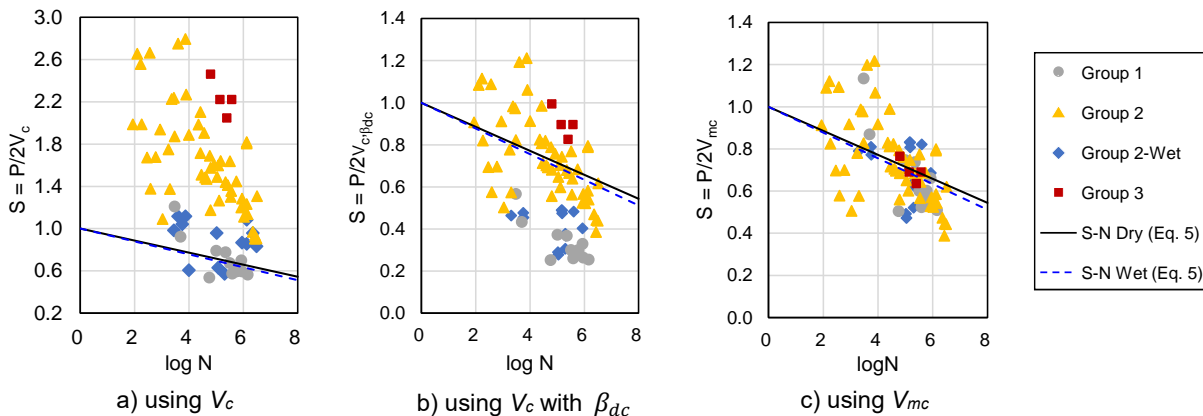


Fig.5 Fatigue life prediction using different shear strength equations

Since previous researchers [18] reported that the RC slabs tested under wet conditions had a shorter fatigue life compared to that of under dry condition, and results of fatigue life prediction of slabs in group 2-wet in Fig. 5b exhibit deviation in the ratio of  $S_{test}/S_{cal}$  with the values of 0.59., the modification factor representing the effect of wet conditions,  $\alpha_{wc}$ , is also proposed in addition to the effect of support conditions in Eq.8.

After the MCFT-based equation was modified and adapted in fatigue life prediction, the results in Fig. 5c demonstrate good agreement between the S-N curve and fatigue life prediction using  $V_{mc}$ . Also, the  $S_{test}/S_{cal}$  ratio in Table 2 becomes very close to 1.00 in all groups. Additionally, further investigation is conducted to confirm the accuracy and reliability of the proposed shear strength equation by comparing it to the existing shear strength equation (Eq. 1). Fig. 6 displays the comparison between shear strength estimated by the proposed equation ( $V_{mc}$ ) (horizontal axis) and existing equation ( $V_{bc}$ ) (vertical axis). Based on the average ratio of  $V_{mc}/V_{bc}$ , which is 1.059 with a slight standard deviation (SD) of 0.149, the proposed equation is reliable in predicting shear strength with good accuracy.

(2) Comparison of proposed equation with existing equation for fatigue life predictions

To validate the proposed shear strength equation for fatigue life prediction, the influence of three key parameters, the main reinforcement ratio ( $\rho_m$ ), the distribution reinforcement ratio ( $\rho_d$ ), and compressive strength of concrete ( $f'_c$ ), are compared throughout the ratio of  $S_{test}/S_{cal}$ . In Fig. 7, the proposed equation incorporating with modified S-N equation proposed by Takeda and Sato [12] by replacing  $V_{bc}$  (Eq. 1) with  $V_{mc}$  (Eq.8) and their equations (hereinafter refer to as Takeda's equation) are evaluated, and their respective distributions for each parameter are depicted. In Fig. 7a, the proposed equation accurately assesses all parameters,

as indicated by regression lines close to zero slope for precise evaluation. Those results are similar to Fig. 7b which shows  $S_{test}/S_{cal}$  distribution obtained from Takeda's equation. This similarity implies that modification of shear strength equation for fatigue life prediction by considering the relatively combined effect of main and distribution reinforcement represented term  $\rho_d/\rho_m$  in  $\beta_{dc}$  of the proposed equation and terms  $\beta_{p1}$  and  $\beta_{p2}$  of Takeda's equation is strongly effective. In addition, Takeda's equation included magnification factor on the compressive strength of concrete, denoted as  $f_{vmcd}$ , meanwhile the proposed equation introduces the coefficient 2.0 in  $\beta_{dc}$  for taking into consideration of enhancement of slab's shear strength by the arching force [14,15] produced by the support confinement. The efficiency of that modification approach is also supported by Fig. 7 on the comparison of  $S_{test}/S_{cal}$  with  $f'_c$ . Eventually, based on the results in Fig. 7. It could be concluded that the proposed equation can be utilized for fatigue life prediction of RC bridge deck slabs subjected to moving wheel loads.

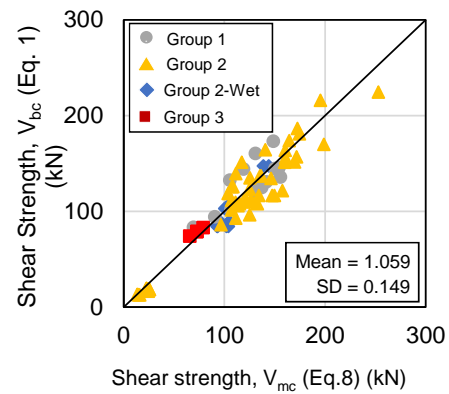


Fig.6 Comparison of shear strength prediction

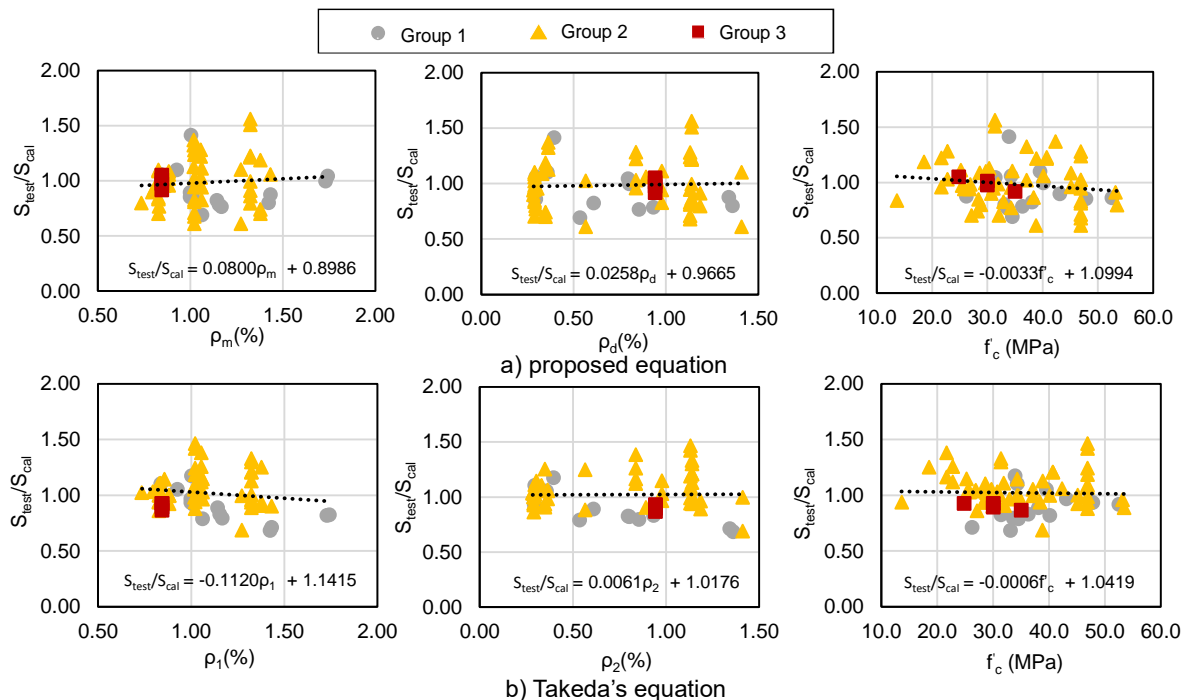


Fig.7 Distribution of  $S_{test}/S_{cal}$ .

#### 4. CONCLUSIONS

- (1) The MCFT-based shear design equation confirms that the fatigue life of RC deck slabs is incredibly impacted by support and moisture conditions as indicated by previous researchers [12]. Therefore, it is necessary to consider these effects when adapting the MCFT-based shear design equation for fatigue life prediction of RC bridge deck slabs.
- (2) The proposed shear strength equation developed by modifying the MCFT-based shear design equation could be used to predict fatigue life of RC slabs failed under moving wheel loading with various support conditions as well as under wet condition. In addition, its reliability and accuracy were validated through the existing equation.
- (3) Based on the comparison of the  $S_{rest}/S_{cal}$  between proposed and existing equations, the results demonstrated that the proposed equation is reliable to adapt in predicting the fatigue life prediction of RC slabs under moving wheel load.

#### ACKNOWLEDGEMENT

The authors acknowledge the funding support provided by JICA-SATREPS for this research.

#### REFERENCES

- [1] Boothby, T. E., and Laman, J. A., "Cumulative Damage to Bridge Concrete Deck Slabs Due to Vehicle Loading," *Journal of Bridge Engineering*, Vol.4, pp. 80-82, Feb. 1999.
- [2] McCarten, P. S., "Bridge deck fatigue: A case for proactive preventive bridge management," in *Bridge Safety, Maintenance, Management, Life-Cycle, Resilience and Sustainability*, CRC Press, Jun. 2022, pp. 449-457.
- [3] Sonoda, K., and Horikawa, T., "Fatigue strength of reinforced concrete slabs under moving loads," *IABSE Reports*, Vol. 37, pp. 456-462. 1982.
- [4] Matsui, S., "Study on Fatigue and Design Method of Concrete Deck Slabs for Road Bridge," *Doctoral dissertation*, Osaka University, 1984.
- [5] Perdikaris, P. C. and Beim, S. R., "RC Bridge Decks Under Pulsating and Moving Load," *Journal of Structural Engineering*, Vol.114, pp. 591-607, Mar. 1988.
- [6] Batchelor, B de V., Hewitt, BE., and Csagoly, P., "An investigation of the fatigue strength of deck slabs of composite steel/concrete bridges," *Transp Res Rec*, Vol.664, pp. 153-161, 1978.
- [7] Maekawa, K., Gebreyouhannes, E., Mishima, T., and An, X., "Three-Dimensional Fatigue Simulation of RC Slabs under Traveling Wheel-Type Loads," *J. of Advanced Concrete Technology*, Vol.4, pp. 445-457, Oct. 2006.
- [8] AASHTO, "AASHTO LRFD bridge design specifications 9th edition," Washington D.C., 2020.
- [9] Fu, G. et al., "Effect of Truck Weight on Bridge Network Costs," Washington D.C., 2003.
- [10] Vecchio, F. J., and Collins, M. P., "The Modified Compression-Field Theory for Reinforced Concrete Elements Subjected to Shear," *ACI Journal*, Vol.83, pp. 219-231, Mar. 1986.
- [11] Bentz, E. C., Vecchio, F. J., and Collins, M. P., "Simplified Modified Compression Field Theory for Calculating Shear Strength of Reinforced Concrete Elements," *ACI Structural Journal*, Vol. 103, pp. 614-624, Jul. 2006.
- [12] Takeda, K., and Sato, Y., "Proposal for fatigue life prediction method for concrete slabs subjected to cyclic wheel-type loads," *Journal of JSCE*, Vol. 79, No. 12, 23-00078, 2023.
- [13] Kongwang, N., Jirawattanasomkul, T., and Sato, Y., "A comparative investigation on shear fatigue strength of RC bridge deck slabs," *Proceedings of the Annual Conference of the Japan Concrete Institute*, Vol.45, pp. 913-918, 2023.
- [14] Keenan, W. A., "Strength and Behavior of Restrained Reinforced Concrete Slabs Under Static and Dynamic Loadings," *Naval Civil Engineering Lab Port Hueneme*, 1969.
- [15] Hon, A., "Compressive Membrane Action in Reinforced Concrete Beam-and-Slab Bridge Decks," *Doctoral dissertation*, Monash University, 2003.
- [16] Furuuchi, H., Takahashi, Y., and Kakuta, Y., "Study on practical design method for shear of reinforced concrete slabs loaded near free edge," *J. of Japan Soc Civ Eng*, Vol. 30, pp. 141-149, 1996.
- [17] Otsuka, Y., Abe, T., and Kida, T., "Experimental study on punching shear strength of reinforced concrete slabs," *Proceeding of Annual academic Conference of the College of Industrial Technology, Chiba, Nihon University*, Vol. 39, No.2, 2006.
- [18] Matsui, S., "Fatigue strength of RC-slabs of highway bridges by wheel running," *Proceedings of the Annual Conference of the Japan Concrete Institute*, Vol.9, No.2, pp. 627-632, 1987.