- Technical Paper -

FLEXURAL BEHAVIOR PREDICTION OF SFRC BEAMS USING FINITE ELEMENT METHOD AND X-RAY IMAGE

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ABSTRACT

This paper presents a novel method for predicting flexural behavior of steel fiber reinforced concrete (SFRC) beams using finite element method and X-ray image. A method to derive the tensile softening curve based on an inversed analysis and X-ray image is proposed. In the finite element (FE) method, the variability of fiber dispersion in individual SFRC beam is considered by determining the different stress-strain relations in each mesh based on the variability of fiber reinforcement identified by X-ray image. The proposed model provides good agreement with the experimental results.

Keywords: SFRC beam, X-ray technique, flexural behavior, distribution of fiber, FE analysis, tension softening curve

1. INTRODUCTION

Steel fiber reinforced concrete (SFRC) is a material characterized by an enhanced post-cracking residual strength due to the crack-bridging fibers. Before cracking, SFRC can be considered as an isotropic material whose tensile behavior is described by the stress-strain relation. After cracking, its tensile behavior due to crack-bridging fibers is described by the stress-crack opening relation.

Therefore, when predicting the flexural behavior of SFRC members, it is necessary to derive the tensile post-cracking strength as an important mechanical property for identifying the stress-crack opening constitutive laws. Due to the difficulties in performing the direct tension test, it is suggested that this mechanical property can be deduced from an indirect method using the bending test of small notched prisms [1,2].

However, experimental researches [3,4] reveal that the tensile post-cracking strength is the material property significantly affected by the distribution and orientation of fibers. Since a number of parameters during the fabrication process cause different fiber dispersions within individual SFRC member, there exists a large scatter of post-cracking responses for the test specimens even if they are reinforced with the same fiber content [5]. This problem causes a difficulty in deriving the mechanical property based on only engineering mechanic theory. Without considering the variability in fiber dispersion, the identification of tensile softening curves provides discrepant results and leads to an overestimation or underestimation of the flexural behavior of SFRC members [6]. Therefore, the variability of steel fiber must be considered in the structural performance assessment of SFRC members [7].

In an effort to make a reliable prediction for flexural behavior of SFRC members, the authors established an analytical method using X-ray images to account for variability of fibers in each member [8]. A concept of deriving a tensile softening curve using the measured fiber distribution properties on X-ray images was introduced. The main objective of the present study is to establish a reliable method for predicting the flexural behavior of SFRC beams using finite element method and X-ray image. A novel procedure for deriving the tensile softening curve using an inversed analysis and X-ray image is presented. In the FE method, the variability of fibers in SFRC beams is considered by determining the different stress-strain relations in each mesh depending on the variability of fiber reinforcement identified by X-ray image. The validity of the proposed method is verified by comparing the computational and experimental results.

2. TEST PROGRAM AND X-RAY PHOTOGRAPHY

2.1 Materials and test specimens

The steel fiber used in this study is 60 mm long and 0.9 mm in diameter (see Table 1). The concrete mixing proportions and quality of materials are listed in Table 2. Table 3 shows the test specimens. For material characterization test, three series of six cylinders and notch prisms are made with different fiber contents of 20 kg/m³, 30 kg/m³, and 40 kg/m³. For the structural test, two beam series are fabricated with fiber contents of 20 kg/m³ and 40 kg/m³.

2.2 Experimental procedure

Regarding the specimen fabrication, the concrete casting procedure for the prism and beam series is

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Table 1 Characteristics of steel fiber						
Fiber code	Shape	L Dia.		Tensile strength		
		(mm)	(mm)	(MPa)		
Dramix 5D	~	60	0.9	2300		

Table 2 Mixing proportions of concrete per m ³						
Water	Cement	F.A.*	C.A. [†]	Fiber	$AE^{\dagger\dagger}$	
(kg)	(kg)	(kg)	(kg)	(kg)	(ml)	
180	360	770	891	20	1800	
180	360	770	888	30	2520	
180	360	770	884	40	2520	

* Fine aggregate with a specific density of 2.60 g/cm³ and a finess modulus of 2.64 g/cm³. [†]Coarse aggregate ($G_{max} \le 15$ mm) with a specific density of 2.64 g/cm³. ^{††}Air entrained agent.

Table 3 Details of test specimen

Specimen	Notation	#	Cross section (mm ²)	Length (mm)	Fiber content (kg/m ³)
	P20	6	150×80	550	20
Prism	P30	6	150×80	550	30
	P40	6	150×80	550	40
Beam	B20	2	140 imes 80	1460	20
	B40	2	140×80	1460	40



Figure 1. Three-point bending test of prism





described in [2]. After filling concrete in the mold, it is externally compacted to prevent the direct contact of vibrator with fibers. X-ray images at the middle parts of each prism and beam are taken, as indicated in Figures 1 and 2. On the 28th day after the casting, the three-point bending test of prisms (see Figure 1) and compression and splitting tests of cylinders are performed. Finally, the four-point bending test of the beam is conducted (see Figure 2).

2.3 Effects of fiber distribution on the flexural behavior of the beam

Figures 3 and 4 show the load-deflection (P- δ) relations and X-ray images of the prism series P20, respectively. Although the six prisms are made using



Figure 3. Load-deflection relations of P20 prism series



the same batch of concrete and fabrication method, it can be seen from Figure 3 that there is a large scatter of post-cracking behaviors among the prisms. This large scatter is caused by the variability of fiber distribution and orientation in each prism at the critical notch plane, as indicated in Figure 4. For example, the prism P20-6 that has the smallest number of fiber at the notch section (see Figure 4 (f)) exhibits lowest post-cracking performance as shown in Figure 3. Whereas, the prism P20-4 that has the largest number of fiber at the notch section (see Figure 4 (d)) provides highest post-cracking performance. These results demonstrate that the flexural post-cracking performance of SFRC members is significantly affected by the fiber distribution at a critical section.

3. FLEXURAL BEHAVIOR PREDICTION OF SFRC BEAMS

3.1 Parameter to account for variability of fibers

The experimental results [8, 9, 10] indicate that the post-cracking performance of SFRC beams depends significantly on the fiber distribution at the critical section. When concrete cracks, the fibers provide the bridging stress between the cracked surfaces by undergoing the pull-out process from concrete. According to the previous researches on the pull-out test of a single fiber from a concrete element [11,12], for a given concrete strength, the effectiveness of a hook-end fiber in countering to the pull-out load depends mainly on its orientation and embedded length in the concrete. Moreover, it was found that fibers at the bottom of the prisms are more effective in resisting to the external stress much more than those at the top [9]. Thus, a parameter Representative Number of Fiber (*RNF*) is proposed to account for the variability of fiber distribution in each SFRC specimen. Firstly, the fiber distribution properties (i.e., orientation α_i , embedded length Le_i, and location Lo_i) of each fiber that lies across the assumed cracking line are measured in each mesh of the prism (see Figure 5). Next, the roughly estimated scores of a fiber pull-out performance S α_i , SLe_i, and SLo_i for *RNF*1 and *RNF*2 in Table 4-6 are assigned to α_i , Le_i, and Lo_i of each fiber. *RNF*1 or *RNF*2 is a summation of the scores of a total number of fibers in each mesh:

$$RNF1$$
 or $RNF2 = \sum_{i=1}^{n} RNF_i = \sum_{i=1}^{n} S\alpha_i \times SLe_i \times SLo_i$ (1)

where

- *RNF*_i : representative number of individual fiber across the assumed cracking line
 - *n* : total number of fibers lying across the assumed cracking line in a mesh.

3.2 Derivation of post-cracking tensile strengths by an inversed FE analysis

In the inverse analysis, a tri-linear stress-crack opening $(\sigma$ -w) curve shown in Figure 6 is used for characterizing the SFRC in tension. The curve is comprised of tri-linear that are defined by four points: (a) tensile strength f_t at the crack width of 0.0 mm, (b) post-cracking tensile strength σ_1 at $w_1 = 0.06$ mm, (c) σ_2 at $w_2 = 0.90$ mm, and (d) zero stress at $w_u = 15$ mm. Therefore, two unknown parameters σ_1 and σ_2 need to be determined for identifying the curve.

Figure 7 illustrates the flow of an inversed analysis to derive the post-cracking tensile strengths from bending test of the notch prisms. In the inversed analysis, the input values of σ_1 and σ_2 are altered until a main criterion is reached: the difference between the areas under computational and experimental curves



Figure 5. Fiber distribution properties measured in a mesh.

is smaller than 5%. When this criterion is satisfied, the values of σ_1 and σ_2 are determined. Note w_1 and w_2 are fixed as 0.06 mm and 0.09 mm while σ_1 and σ_2 are altered in the inverse analysis since w_1 and w_2 were slightly changed while maintaining good fits of two important points on most experimental curves: point (e), the minimum load after the first cracking, and point (f), the maximum load.



Figure 6. Tri-linear σ -w tension softening curve







Figure 8. Tri-linear σ -w curves and σ_1 and σ_2 obtained from the inverse analysis

Table 4. Assumed score for the orientation of fiber (for RNF1 and RNF2)									
Orientation,	α 0°	0°~20°	20°~30°	30°~40°	40°~50°	50°~60°	60°~70°	° 70°~80°	80°~90°
Score, Sa	0.98	0.94	0.87	0.77	0.64	0.50	0.34	0.17	0.01
Table 5. Assumed score for the location and embedded length of fiber (for <i>RNF</i> 1)									
Embed	Embedded length, Le (mm) $0\sim10$ $10\sim30$		Location, I	Lo (mm)	$0 \sim h sp/8$ h	sp/8~hsp/4 h	nsp/4~hsp		
	Score, SLe		0.1	1.0	Score,	SLo	1.0	0.25	0.0
Table 6. Assumed score the location and embedded length of fiber (for <i>RNF</i> 2)									
E	mbedded lengtl	n, Le (mm)	0~10	10~30	Location,	Lo (mm)	0~hsp/2	hsp/2~h	sp
	Score, S	Le	5.0	1.0	Score,	SLo	1.0	0.0	

As the result of the inversed analysis, a number of different tensile softening curves with various values of σ_1 and σ_2 are obtained, as shown in Figure 8. It can be inferred that the differences in these curves result from the distinctive experimental P- δ responses of each SFRC prism that are affected by the variability in fiber distribution and orientation.

3.3 Relationship between *RNF* and post-cracking tensile strengths

The relationship between post-cracking tensile strengths (σ_1 and σ_2) and the fiber distribution parameters *RNF* (*RNF*1 and *RNF*2) are established as shown in Figures 9 and 10. The relationship suggests a good relationship between *RNF* and post-cracking tensile strengths. Through these relationships, σ_1 and σ_2 can be derived using *RNF* as the input once the fiber distribution properties are quantified by the X-ray image. Hence, the variability of fiber dispersion in each SFRC member can be considered in the derivation of post-cracking tensile strengths and identification of the tensile softening curve.

3.4 FE analysis of the beam

Figure 11 illustrates a two-dimensional finite element model used to simulate the flexural response of SFRC beam. The square mesh size (35 mm × 35 mm) is chosen for concrete element. The concrete is modeled using eight-node plane stress element, and the smeared crack model is used for the increment of stress-strain relation of concrete. To perform the FE analysis of the beam, the tri-linear stress-crack opening (σ -w) curve is first determined in each mesh using the numerical values of *RNF*1 and *RNF*2 that are determined from

X-ray images, and each σ -w curve is then converted into stress-strain (σ - ε) relations (see Figure 12). The conversion from a crack opening to a tensile strain can be achieved using the structural characteristic length l_{cs} . The purpose of using l_{cs} is to connect a continuum mechanic, governed by σ - ε constitutive relation, and fracture mechanics, governed by a stress-crack opening (σ -w) law [7]. It allows the transformation of a crack opening to an equivalent tensile strain via a relationship $\varepsilon = w/l_{cs}$. The characteristic length is chosen herein as the square root of the mesh (i.e., 35 mm), according to [13]. Therefore, the strains ε_t , ε_1 , ε_2 , and ε_u shown in Figure 13 can be determined using the following formula:

$$\varepsilon_t = f_t / E \tag{2}$$

$$\varepsilon_1 = w_1 / l_{cs} + \varepsilon_t \tag{3}$$

$$\varepsilon_2 = w_2 / l_{cs} + \varepsilon_t \tag{4}$$

$$\varepsilon_u = w_u / l_{cs} + \varepsilon_t \tag{5}$$

3.5 Effects of different mesh patterns

Since fiber's distribution in each mesh is changed due to the mesh patterns, different mesh patterns can affect *RNF* or stress-strain relations in each mesh and results in different behavior. Thus, the effect of four mesh patterns on the result of FE analysis is examined. As shown in Figure 14, the mesh pattern 2 moves from the location of mesh pattern 1 to the left side by a half of mesh length. Whereas, the patterns 3 and 4 move from the location of mesh pattern 1 to the right and left sides by a quarter of mesh length, respectively.



Figure 12. Identification of stress-strain relations in each mesh using numerical values of *RNF*

Figure 13. Stress-strain relation of SFRC



mesh patterns for the beam B20-1

3.6 Validity of the proposed method

The validity of the proposed method is verified by comparing the cracking location and P- δ response predicted by the FE analysis with those of the test beams, as shown in Figures 15, 16, and 17, respectively. Figure 15 shows the predicted cracking locations by the FE analysis of the beams B20-1 and B40-2



depend on the mesh patterns. The location of primary crack might represent a critical plane in which the elements have the weakest post-cracking tensile strengths. Figure 15 indicates that the prediction accuracy of primary cracking location for beam B20-1 is better than that of the beam B40-2. Moreover, From Figure 16, it can be seen that the flexural response of the beam B20-1 depends on mesh patterns that were modeled; and the mesh pattern 4 and pattern 2 represent the lower and upper bounds among the four mesh patterns.

Figure 17 shows the P- δ relations of simulated beams by FE analysis versus those of the test beams. The gray band curve represents an upper and lower bounds of P- δ responses estimated by the FE analysis using the four different mesh patterns. While, the dash curve is the P- δ response computed using only the average tension softening curve from a series of six notch prisms. The other curve (i.e. dashed-dotted lines) is the computed P- δ response for the case that the cracking section of a beam is known after the bending test. In this case, the σ - ε relations are identified using the values of RNF for only one column of the four meshes at the location where a cracking path of the test beam occurred while keeping other σ - ε relations for the rest of meshes in elastic. The agreement between the analytical and experimental P-8 responses is very satisfactory, except for the beam B20-2 in which the FE analysis overestimates that of the test beam. Nevertheless, it should be noted that the FE model which considers the variability of fibers using X-ray image yields better results than the FE model using only the average tensile softening curve.

Moreover, the FE analysis using the actual cracking location provides better conservative results than others. This means that if the cracking location can be predicted more correctly based on the X-ray image, it can help to conduct the FE analysis more precisely.

4. CONCLUSIONS

In this paper, a flexural prediction method of SFRC beams using FE analysis and X-ray technique has been presented, and its validity is also verified. The following points can be concluded:

- (1) A novel approach to derive post-cracking tensile strengths by using an inversed analysis and a proposed parameter *RNF* quantified by the X-ray image was established. This approach provides a rational means for an identification of the tension softening curve while considering the variability of fibers in each SFRC specimen.
- (2) The proposed model taking into account the variability of fibers based on the X-ray image provides good agreement with the experimental results.

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