- Technical Paper -

SHEAR BEHAVIOR OF TAPERED RC BEAMS WITH STIRRUPS

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ABSTRACT

This paper presents the experiment of four tapered RC beams with and without stirrups. By comparing with tapered RC beams without stirrups, the shear resistance mechanism of the beams with stirrups was clarified. The location of the critical section was found to be increased due to the use of stirrups, which was related to the stirrup ratios as well. The evaluation method of shear carried by stirrups was also proposed by using the compression strut inclination and critical section at the occurrence of the main diagonal crack. The calculated value showed good correspondence with the experimental results.

Keywords: shear, tapered RC beam, critical section, stirrup ratio, inclination of compressive strut

1. INTRODUCTION

Reinforced concrete (RC) members with the variable depth of the cross section along its axis are wildly used in structural portal frames, cantilevers and bridge structures. This kind of members can be designed according to the required resistance against the external load. The tapered RC beam in the Fig. 1 [1] has larger depth at the middle part to resist large flexural moment. Consequently, it can efficiently use the concrete and steel reinforcements, considerably reducing the structure's weight, and contribute to an aesthetic design. However, it is insufficient of the experimental data to predict the shear behavior of tapered RC beams. Moreover, rational and economical design method for RC members with variable depth in the JSCE specifications for concrete [2] has not been completed. Engineers are using such beams based on the empirical and conservative design method which is not accurate and uneconomic. Therefore, it is necessary to propose a convenient evaluation method with high accuracy and compatibility for tapered RC beams to ensure the reasonable design.

According to the recent research of the authors [3], the shear resistance mechanism of tapered RC beams ($2.5<a/d<5.0$) without stirrups was clarified. The new evaluation method for the shear capacity of tapered RC beams without stirrups was also proposed in the previous research [3]. Since stirrups are normally used in the concrete structures, the experiment of three tapered RC beams with stirrups was conducted in this study. As the elongation of the previous research, the objective of this study is to propose an evaluation method for shear capacity of tapered RC beams with stirrups.

Fig. 1 Tapered RC beam in the pier structure [1]

2. EXPERIMENTAL PROGRAMS

2.1 Test Specimens and Materials

The details of tested beams are illustrated in Fig. 2 and Table 1, including the dimension and reinforcing bars arrangement of the beams. Totally three tapered RC beams with stirrups and one tapered RC beam without stirrups were used in this paper. All the specimens were designed to fail in shear. The cross sections at the loading point and support were fixed, while the taper slopes and stirrup ratios were varied by changing the shear span's length. Therefore, the specimens were named by the shear span's length. For example, in the beam VS650, "V" means variable effective depth, "S" means with stirrups and "650" represents the shear span. By comparing the tapered RC beams with and without stirrups in same dimensions, the effect of stirrups on shear behaviors could be clarified.

Two longitudinal D22 tensile bars (A_s=380.1mm^2) with yield strength of 930 N/mm^2 were used in all four specimens. In the specimens of V650 and VS650, D10 stirrups (A_s=71.33mm^2) with yield strength of 295 N/mm^2 were used, while in the other two specimens, D6 stirrups (A_s=31.67mm^2) with yield strength of 295 N/mm^2 were used to get different stirrup ratios.

2.2 Loading Test and Instrumentation

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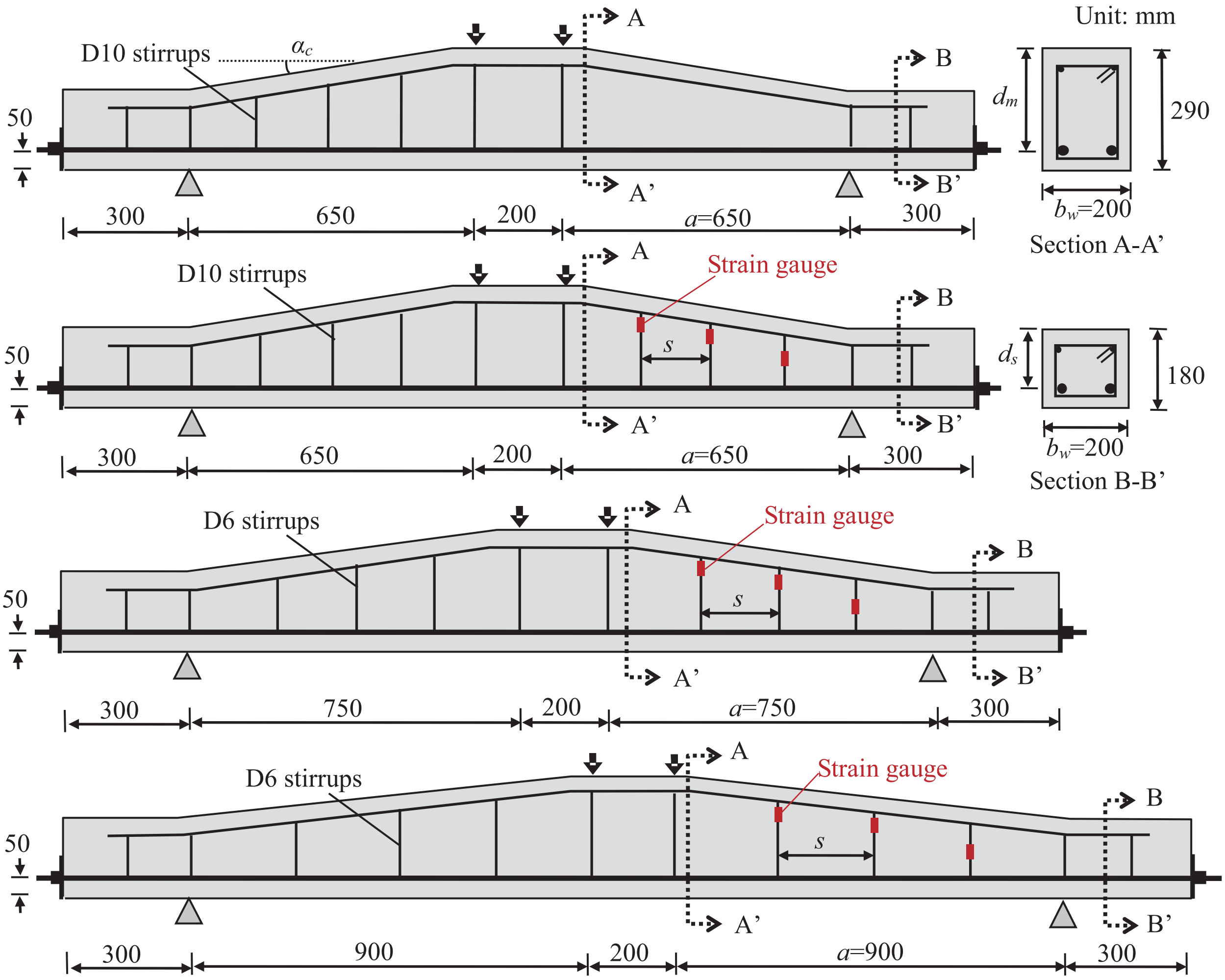

Fig. 2 Specimen dimensions

Table 1 Specimens' details and material properties

No.	α_c (°)	f_c' (N/mm^2)	a (mm)	d_s (mm)	d_m (mm)	a/d_m	s (mm)	r_w (%)	A_s (mm^2)	b_w (mm)	V_{exp} (kN)
V650	10.4	34.4	650	130	240	2.71	-	0	760.2	200	75.8
VS650	10.4	44.7					162.5	0.44			152.2
VS750	8.9	41.0	750			3.13	187.5	0.17			118.0
VS900	7.4	36.8	900			3.75	225.0	0.14			108.5

α_c: taper slope; f_c': compressive strength of concrete; a: shear span; d_s: effective depth at support; d_m: effective depth at mid span; s: stirrup spacing; r_w: stirrup ratio; A_s: cross section area of tensile reinforcement bars; b_w: width of the beam; V_{exp}: shear capacity from the experiments.

A four-point bending test with simply-supported condition was provided to all specimens as illustrated in Fig. 2. Steel plates of 50 mm width were placed on the pin-hinge supports, while teflon sheets and grease were inserted between the specimen and supports in order to prevent the horizontal friction. At the loading points, steel plates with 65 mm width and 200 mm length were also placed.

During the loading tests, the mid-span deflection was measured using four displacement transducers at the mid span and supporting points. Based on the main diagonal crack occurred in the tapered RC beam without stirrups in the authors' previous experiments [3], the strain gauges were attached in different locations for each stirrup. For the stirrup near the support, the strain gauge was at the middle of the stirrup, while for the other two stirrups, the strain gauges were 20 mm from the top of stirrups (Fig. 2). In addition, the crack propagation on the surface of shear span during the loading test was captured by taking pictures.

3. EXPERIMENTAL RESULTS AND DISCUSSIONS

3.1 Crack Patterns and Load-displacement Curves

Figure 3 shows the crack patterns of the beam V650 and VS650 with the load-deflection curves. From these results, it was found that the shear capacity was increased due to the using of the stirrups. The main diagonal crack in VS650 was almost same with that in V650, which indicates the same shear resistance mechanism at least until the occurrence of diagonal cracks in these two beams. However, since there were stirrups in the shear span of VS650, the diagonal crack developed towards the loading point. Finally the shear failure happened near the loading point.

Figure 4 shows the load-deflection curves of all the specimens with stirrups, while Fig. 5 shows the crack patterns of these beams just before and after the peak. The shape of main diagonal cracks in these beams also showed the similarity with that of the tapered beam without stirrups. The differences were the different failure locations in each beam: the failure of V650 came with the occurrence of the diagonal crack, while the main diagonal cracks in the beams with stirrups developed towards the loading point and failed with the penetrating of concrete section by shear cracks at different locations due to the different stirrup ratios.

3.2 Evaluation method for the shear carried by the concrete

Cracks in the shear span (Fig. 5) caused the tensile strains in the three stirrups of each beam, while the stirrups strengthened the whole concrete beam. In order to evaluate such shear contribution properly, it is assumed that all the three stirrups with tensile strains make the shear contributions in the experiments. By summing the shear contributions of the three stirrups in the shear span from the strain value obtained from the

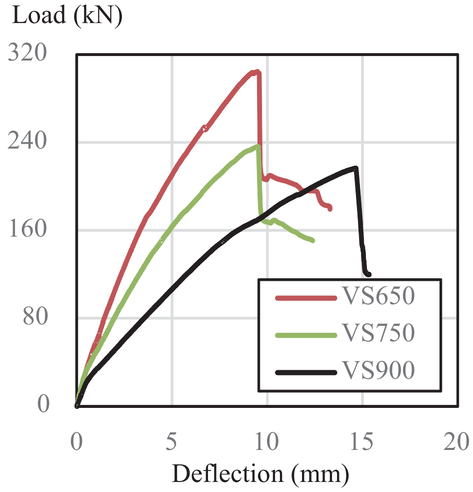

Fig. 4 Load-deflection curves of specimens with stirrups

strain gauges on the stirrups (Fig. 2), the contribution of the shear carried by stirrups in these three beams were shown in Fig. 6. The similar curves indicate the similar situations in these beams that the stirrups started to carry the shear force after the occurrence of diagonal cracks. However, since the main diagonal crack only passed one stirrup near the support completely, the

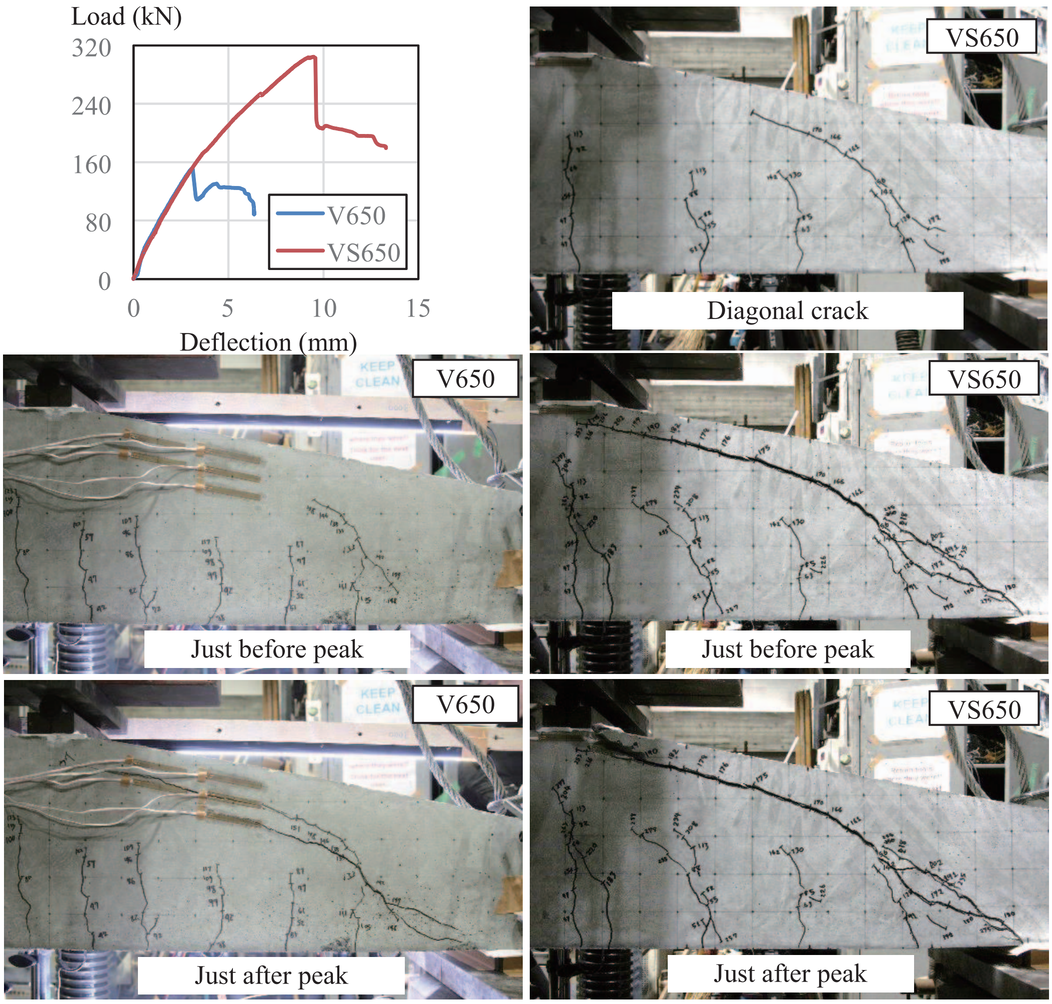

Fig. 3 Experimental results for the beams V650 and VS650

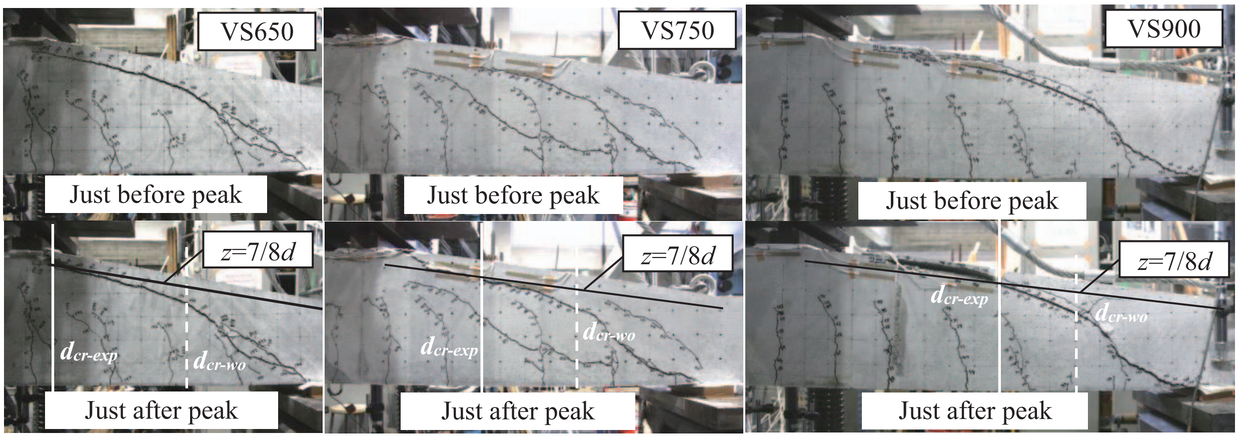

Fig. 5 Crack patterns of specimens with stirrups

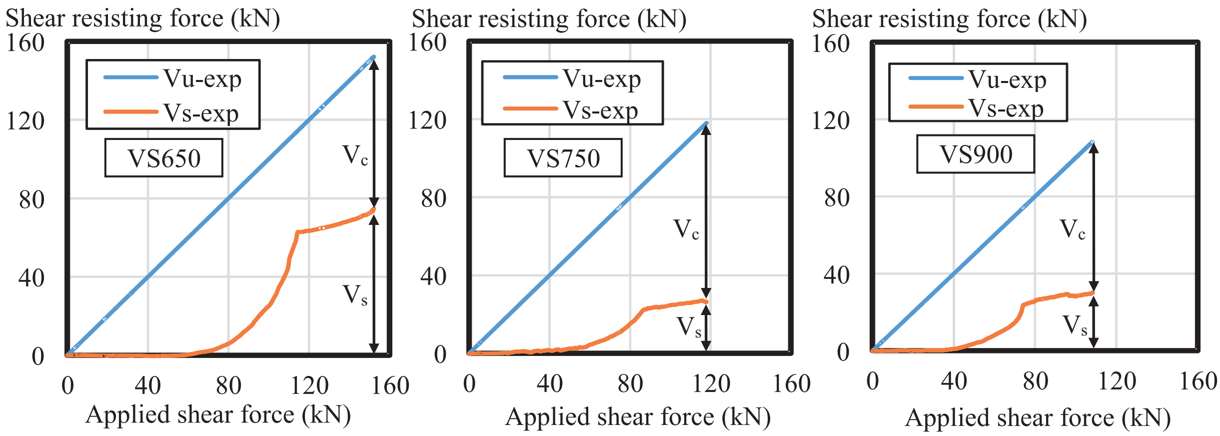

Fig. 6 Shear carried by stirrups

Table 2 Shear contributions of stirrups and concrete

No.	$V_{u\text{-}exp}$ (kN)	$V_{s\text{-}exp}$ (kN)	$V_{c\text{-}exp}$ (kN)
VS650	152.2	74.7	77.5
VS750	118.0	26.5	91.5
VS900	108.5	30.1	78.4

$V_{u\text{-}exp}$: shear capacity from the experiments; $V_{s\text{-}exp}$: shear carried by stirrups from the experiments; $V_{c\text{-}exp}$: shear carried by concrete, equals to $V_{u\text{-}exp}$-$V_{s\text{-}exp}$.

other two stirrups were not yielded at the peak. Therefore, after the one stirrup that the diagonal crack passed was yielded, the shear force carried by stirrups did not increase very much. By subtracting the shear carried by stirrups from the shear capacity, the shear carried by the concrete could be obtained in each beam, which is shown in Table2.

From the previous research of the authors [3], the evaluation method for the shear carried the concrete V_c when no stirrups were used was proposed. The basic concept is shown in Fig. 7. By considering the contribution of the inclined compressive force in the concrete, the force equilibrium and the geometry relations, the shear capacity of tapered RC beams without stirrups can be calculated as the following equations:

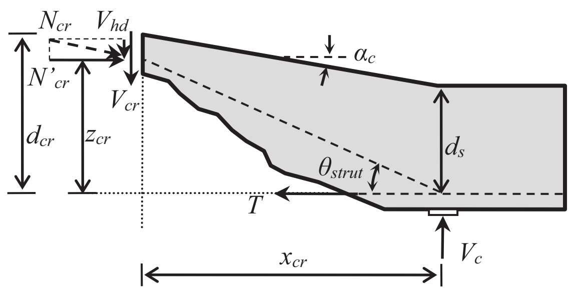

Fig. 7 Forces acting in tapered RC beams [3]

$$V_c = V_{cr} + V_{hd} \quad (1)$$

$$V_{hd} = N'_{cr} \tan\alpha_c \quad (2)$$

$$N'_{cr} = M_{cr} / z_{cr} = 8V_c x_{cr} / 7d_{cr} \quad (3)$$

$$x_{cr} = (d_{cr} - d_s) / \tan\alpha_c \quad (4)$$

$$V_c = 7V_{cr} / (8d_s / d_{cr} - 1) \quad (5)$$

Where, V_c is the shear capacity of tapered RC beam without stirrups, V_{cr} is the shear carried by the critical section which uses the effective depth d_{cr} of critical section, V_{hd} is the vertical component of the compression force N_{cr}, x_{cr} is the distance from support to the critical section, z_{cr} is the internal lever arm (=jd_{cr}) with j = 7/8, and α_c is the slope of the taper. Eq. (5) is obtained by substituting Eqs. (2)-(4) into Eq. (1) [3]. For the evaluation method of V_{cr}, by substituting the value of d_{cr} into the equations by Niwa et al. [4] (Eq.

(6) and (7)) which was slightly modified by JSCE and adopted in the standard specifications [2] for design shear capacity of linear members without shear reinforcing steel, the shear carried by the critical section can be calculated.

$$V_{cr} = \alpha f_c'^{1/3} p_w^{1/3} \left(\frac{1000}{d_{cr}}\right)^{1/4} b_w d_{cr} \quad (6)$$

$$\alpha = 0.2\left(0.75 + \frac{1.4}{a/d_{cr}}\right) \quad (7)$$

Basically, the cross section in which the shear failure occurred is defined as the critical section. For the effective depth of the critical section when no stirrups were used or $d_{cr\text{-}wo}$, since the failure happens with the sudden occurrence of the diagonal crack, and the occurring section is almost same as the section where the compressive force changes direction, it is used as the critical section when no stirrups were used. The determining method with Eqs. (8)-(10) were also proposed through FEM analysis in the previous research [3], where θ_{strut} is the inclination of the compressive strut:

$$\tan\theta_{strut} = 0.75\tan\alpha_c + 0.409 \quad (8)$$

$$d_{cr\text{-wo}}/d_s = \tan\theta_{strut}/(\tan\theta_{strut} - 0.875\tan\alpha_c) \quad (9)$$

$$d_{cr\text{-wo}}/d_s = (6\tan\alpha_c + 3.27)/(3.27 - \tan\alpha_c) \quad (10)$$

However, in tapered RC beams with stirrups, the shear failure happens with the penetrating of concrete section by shear cracks based on the slow occurrence of the diagonal crack. The cracks made the concrete's compressive strength reduced [5]. Therefore, the assumption was made that the section where the main diagonal crack or shear crack passed the compressive force line (z=7/8d) is the critical section. Since the shear resistance mechanism of the concrete was considered to be similar in both RC tapered beams with or without stirrups, the above evaluation method was supposed to be applicable for RC tapered beams with stirrups as well, by using the effective depth of the critical section $d_{cr\text{-}w}$ in beams with stirrups.

Figure 5 and Table 3 show the effective depth $d_{cr\text{-}exp}$ of critical section from the experiments and $d_{cr\text{-}wo}$ from the calculation based on the specimen dimensions. The white solid lines mean the critical sections of tapered beams with stirrups, where the main diagonal cracks or shear cracks passed the compressive force line (z=7/8d). The white dashed lines mean the original critical sections if the stirrups were not provided in these tapered RC beams. By fitting the linear relationship in Fig. 8, it was found that the larger of the stirrup ratio, the more increase of the effective depth from $d_{cr\text{-}wo}$ to $d_{cr\text{-}exp}$. By simplifying the linear relationship for calculation and consistency, the equation was derived as:

$$d_{cr\text{-w}} = d_{cr\text{-wo}}(0.677 r_w + 1) \quad (d_s \le d_{cr\text{-}w} \le d_m) \quad (11)$$

Table 3 Value of effective depth of the critical sections

No.	$d_{cr\text{-}wo}$ (mm)	$d_{cr\text{-}exp}$ (mm)	ratio	r_w (%)	$d_{cr\text{-}w}$ (mm)
V650	184.0	184.0	1.000	0	184.0
VS650	184.0	240.0	1.304	0.44	238.7
VS750	175.9	205.2	1.166	0.17	196.0
VS900	167.5	186.8	1.115	0.14	183.5

$d_{cr\text{-}wo}$: effective depth calculated from Eq. (10); $d_{cr\text{-}exp}$: effective depth obtained from the experiments; ratio: value of $d_{cr\text{-}exp}/d_{cr\text{-}wo}$; $d_{cr\text{-}w}$: effective depth calculated from Eq. (11).

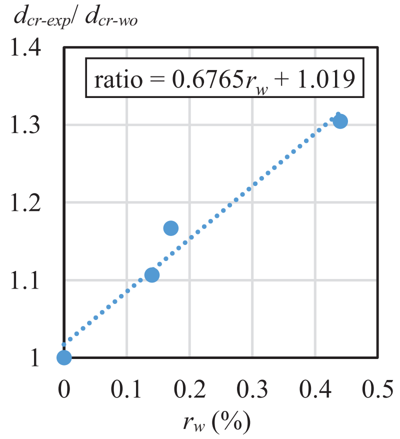

Fig. 8 Relationship between stirrup ratio and the ratio of effective depths

Table 4 shows the calculation results of shear carried by the concrete with the effective depth of $d_{cr\text{-}exp}$ and $d_{cr\text{-}w}$, all of which show a relatively good agreement with the experimental results. For the beam VS650, since the failure happened near the loading point, the contribution of the inclined compressive force in the concrete did not exist anymore. Therefore, the shear capacity was calculated only using the equations of Niwa et al. [4] (Eq. (6) and (7)). For the beams VS750 and VS900, the proposed method in this study was used. Even comparing with the shear capacity calculated with

Table 4 Summary of calculations for shear contributions of concrete

No.	f_c' (N/mm^2)	$V_{c\text{-}exp}$ (kN)	$V_{c\text{-}jsce}$ (kN)	$d_{cr\text{-}exp}$ (mm)	$V'_{c\text{-}cal}$ (kN)	$V_{c\text{-}exp}/V'_{c\text{-}cal}$	$d_{cr\text{-}w}$ (mm)	$V_{c\text{-}cal}$ (kN)	$V_{c\text{-}exp}/V_{c\text{-}cal}$
VS650	44.7	77.5	71.9	240.0	71.9	1.078	238.7	71.6	1.082
VS750	41.0	91.5	66.0	198.9	94.7	0.966	196.0	91.8	0.997
VS900	36.8	78.4	59.7	186.8	76.9	1.019	183.5	74.3	1.055

$V_{c\text{-}jsce}$: calculated shear contribution by concrete with current JSCE method (Eq. (6) and (7)) and maximum effective depth d_m; $V'_{c\text{-}cal}$: calculated shear contribution by concrete with the value of $d_{cr\text{-}exp}$; $V_{c\text{-}cal}$: calculated shear contribution by concrete with the value of $d_{cr\text{-}w}$.

Table 5 Summary of calculations for shear contributions of stirrups

No.	V'_s (kN)	A_w (mm^2)	$d_{cr\text{-}wo}$ (mm)	θ_{strut} (º)	s (mm)	$V_{s\text{-}cal}$ (kN)	$V_{s\text{-}exp}$ (kN)	$V_{c\text{-}cal}$ (kN)	$V_{c\text{-}exp}$ (kN)	$V_{u\text{-}cal}$ (kN)	$V_{u\text{-}exp}$ (kN)	$V_{u\text{-}exp}/V_{u\text{-}cal}$
VS650	54.4	142.7	184.0	28.66	162.5	76.3	74.7	71.6	77.5	147.9	152.2	1.029
VS750	20.9	63.3	175.9	27.78	187.5	29.1	26.5	91.8	91.5	120.9	118.0	0.976
VS900	17.4	63.3	167.5	26.84	225.0	24.0	30.1	74.3	78.4	98.3	108.5	1.104

V'_s: calculated shear contribution by stirrups with the normal method (θ equals to 45º) and maximum effective depth d_m; θ_{strut}: compressive strut inclination calculated from Eq. (8); $V_{u\text{-}cal}$: value of $V_{c\text{-}cal} + V_{s\text{-}cal}$.

current JSCE method and maximum effective depth d_m, the proposed method is more accurate. Therefore, it is acceptable to use the equations of Niwa et al. to evaluate the shear carried by the concrete, when $d_{cr\text{-}w}$ is almost same as $d_{m.}$ For the other situations, the proposed method in this paper can be adopted.

3.3 Evaluation method for the shear carried by the stirrups

From the location of the main diagonal crack (Fig. 5) and the strain values of strain gauges attached on the stirrups, the main shear contribution of stirrups was made by the stirrups near the support. Just as shown in Fig. 9 as one example, since the main diagonal crack only passed the stirrup near the support completely, the other stirrups were not yielded at the peak. Considering the fact that, the shapes of the cracks were almost same in the RC tapered beams with or without stirrups at the occurrence of the diagonal cracks, and the main diagonal crack did not cause more stirrups yielded after that, one new assumption was proposed for the evaluation method of shear carried by stirrups. The effective depth of critical section $d_{cr\text{-}wo}$ and the compressive strut inclination θ_{strut} obtained from Eqs. (8)-(10) for the RC tapered beam without stirrups were used to evaluate the shear carried by stirrups for the RC tapered beams with stirrups in the same dimensions:

$$V_s = A_w f_{wy} \left(z \cot\theta_{\text{strut}} / s\right) \quad (12)$$

$$z = 0.875 d_{cr-wo} \quad (13)$$

Where, A_w is the cross sectional area of stirrups in the range of s, f_{wy} is the yield strength of stirrup, z is the internal lever arm. Table 5 shows the calculation results of shear carried by the stirrups using the above evaluation method as well as the total shear capacity. Even comparing with the results obtained from normal method (θ equals to 45º) and maximum effective depth d_m, it is still showing a relatively good agreement. Therefore, it is acceptable to use the proposed evaluation method in this study to evaluate the shear carried by the stirrups for the tapered RC beams with stirrups. However, since the experimental data is not sufficient, more researches are needed to improve the proposed method and verify the validity.

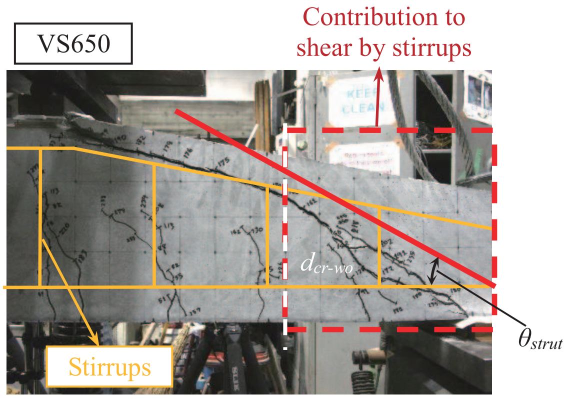

Fig.9 The locations of the main diagonal crack and stirrups

4. CONCLUSIONS

(1) The shear resistance mechanism of concrete in RC tapered beams with stirrups was similar with that in RC tapered beams without stirrups, while the critical section where the shear failure occurred became larger due to the using of stirrups.

(2) The larger of the stirrup ratio, the more increase of the critical section. The simple linear relationship between stirrup ratio and effective depth was also proposed, with which the shear carried by concrete can be predicted.

(3) By analyzing the crack patterns, the new method was proposed to evaluate the shear carried stirrups as well, and the calculated shear capacity using the above evaluation method showed a reasonable agreement with the experimental results.

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