- Technical Paper -

PREDICTION OF SHEAR STRENGTH OF HIGH-STRENGTH CONCRETE MEMBERS WITHOUT WEB REINFORCEMENT

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ABSTRACT

This paper describes the prediction of diagonal cracking shear strength of reinforced high-strength concrete (HSC) members without web reinforcement. It was found that the diagonal cracking shear strength of HSC is governed by the ratio of uniaxial compressive strength to tensile strength (i.e., its ductility number, DN) of the concrete relative to that of the aggregate. By considering aggregate and concrete strengths, shear strength prediction equations and concrete strength regions were suggested. The proposed equations were found to be reliable for design purposes.

Keywords: aggregate strength, concrete strength, high-strength concrete, shear strength

1. INTRODUCTION

High-strength concrete (HSC) is being increasingly used in buildings and bridges because it enables the use of smaller cross-sections, longer spans, reduction in girder height and improved durability [1]. Presently the target compressive strength of concrete easily exceeds 100 MPa. However, the use of HSC has led to some concerns about its diagonal cracking shear strength since the shear strength does not increase as expected with the increase in the compressive strength of concrete [2, 3].

Concrete is a 3-phase composite material composed of cement paste, aggregate and an aggregate/cement paste interface. In HSC, the hardened cement paste and the transition zone between the cement paste and coarse aggregate seldom become strength-limiting because HSC typically corresponds to water-cement ratios (*w*/*c*) in the order of 0.2 to 0.3. Therefore, the crack surface of HSC members is relatively smooth when compared with normal-strength concrete (NSC) because cracks penetrate through the aggregate [2]. However, in NSC concrete mixtures, which are made with a high *w*/*c* ratio (0.4 to 0.7), the weakest components are the hardened cement paste and the transition zone rather than the properties of coarse aggregate [4, 5]. Therefore, when designing NSC mixtures, the properties of aggregate are rarely a matter of concern. However, a past study found that the ratio of uniaxial compressive strength to tensile strength (that is, the ductility number, or DN) of the concrete relative to that of the aggregate governs the diagonal cracking shear strength of HSC [2].

Most theoretical and experimental studies to date have concentrated on beams without web reinforcement since it is generally accepted that they exhibit typical brittle failure and also demonstrate a significant size effect [6]. For slender RC beams without web reinforcement where shear span to depth ratio (*a*/*d*) is greater than 2.5, the shear force is carried by: 1) the shear resistance of uncracked concrete in the compression zone; 2) the interlocking action of aggregate along the rough concrete surfaces on each side of a crack; and, 3) the dowel action of the longitudinal reinforcement. In rectangular beams, the proportion of the shear force carried by these mechanisms is as follows: 53-90% by the uncracked concrete in the compression zone and through aggregate interlocking, and 15-25% by dowel action [7]. Therefore, the diagonal cracking shear strength of RC members strongly depends on the strength of concrete both compressive strength and tensile strength [8-10]. Also, aggregate strength controls the concrete strength, particularly of HSC [9, 10].

During the last 50 years, a lot of experimental research work has looked at the shear design of reinforced concrete (RC) members. The differences between diagonal cracking shear strength expressions proposed by different investigators and code provisions are due to a considerable scatter of experimentally observed shear strengths of HSC members [2, 3]. Until now, no research has attempted to explain this scatter with regard to concrete and aggregate strengths.

The rapidly increasing use of HSC is outpacing the development of appropriate recommendations for diagonal cracking shear strength. Therefore, the objective of this study is to propose a simple and accurate equation predicting the diagonal cracking shear strength of RC members without web reinforcement with respect to aggregate and concrete strengths. In addition, a simplified equation is also proposed for design purposes. These equations are then

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compared with other prediction equations for diagonal cracking shear strength of beams without web reinforcement.

2. DIAGONAL CRACKING SHEAR BEHAVIOR

2.1 Strength Variation in Rocks

The majority of coarse aggregate used in concrete is derived from sedimentary and igneous rocks [9, 11]. The uniaxial compressive strength (σ_c) and Brazilian splitting tensile strength (σ_t) of rock are dependent on the location where it is mined. For example, the uniaxial compressive strengths of granite (igneous rock) from Ibaraki (Japan) and Boston (USA) are 285 MPa and 75 MPa respectively; and, for limestone (sedimentary rock) from Colorado (USA) and Ontario (Canada) are 142 MPa and 64 MPa, respectively (Table 1) [5, 10, 12]. Further, the strength anisotropy of an individual rock is affected by the shape-preferred (plane) orientation of rock-forming minerals [9, 10, 13]. Therefore, the two strength measures (σ_c, σ_t) have maximum and minimum values that depend on the orientation of planes in a given rock sample. Considering this, it is especially important to consider aggregate strengths in HSC.

According to past studies, the DN (σ_c/σ_t) can be used as a measure of concrete brittleness since it governs the material friction angle (φ) [2, 10]. Also, a higher value of DN corresponds to a more brittle concrete [10].

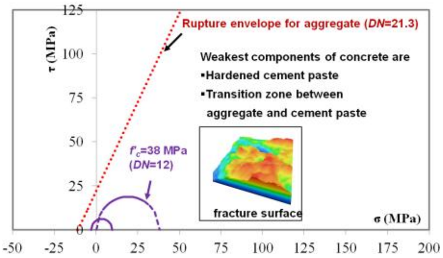
Table 1 Strength variation in rocks [5, 10, 12]

location/ country	Rock type	σ_c (MPa)	σ_t (MPa)
Ibaraki, Japan	Granite	285	15.3
Fujian, China		150	14.0
CA, USA		165	8.9
Boston, USA		75	4.6
Neungdong, South Korea		239	12.4
Soel, South Korea		177	12.0
Manapouri, New Zealand	Granitic orthogneiss	163	6.3
Hunan, China	Limestone	100	11
Colorado, USA		142	6.7
Ontario, Canada		64	5.7
West Side Sewage Tunnel, USA	Basalt	205	9.6

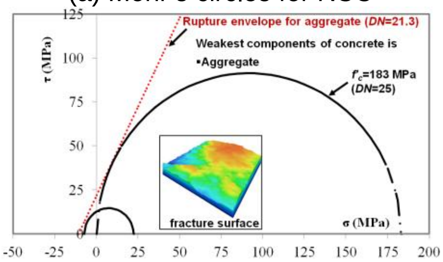
2.2 Effect of DN on Diagonal Cracking Shear Behavior of RC Members

After cracking, shear is resisted by aggregate interlock, the dowel action of tension reinforcement bars, and resistance provided by uncracked concrete in the compression zone of the beam. The percentage carried by aggregate interlock and uncracked concrete in the compression zone depends strongly on the surface roughness at the crack as well as concrete brittleness. According to Perera and Mutsuyoshi [2, 10], the ductility number of the aggregate (DNA) relative to that of concrete governs the fracture surface roughness and brittleness of concrete.

When Mohr's circle of NSC strengths [Fig.1 (a) f'_c=38 MPa] was under the rupture envelope of aggregate, the weakest components were the hardened cement paste and the transition zone between the cement paste and coarse aggregate rather than the strength of coarse aggregate. That is, the fracture surface was rough as cracks did not penetrate the aggregate. However, when Mohr's circles of HSC strengths [Fig.1 (b) f'_c=183 MPa) reached the rupture envelope of aggregate, the weakest component was the strength of coarse aggregate. Therefore, aggregate in HSC ruptured with a smooth fracture surface. That is, the fracture surface of HSC was smoother than that of NSC because cracks penetrated the aggregate. Further, a higher DN in HSC corresponds to a more brittle concrete.



(a) Mohr's circles for NSC



(b) Mohr's circles for HSC

Fig.1 Mohr's circles for rock and concrete (f'_c is compressive strength of concrete) [10]

2.3 Shear Behavior

Perera and Mutsuyoshi [2, 10] proposed the following diagonal cracking shear behavior with respect to DNs of concrete and aggregate: When the *DN* of concrete was lower than that of the aggregate, the diagonal cracking shear strength increased with the increase of concrete strength due to rough fracture surface and increased tensile strength. When the *DN*s of the concrete and aggregate were equal, shear strength stayed constant at the maximum value. However, when concrete had a higher *DN* than the aggregate, diagonal cracking shear strength decreased due to the smooth fracture surface and high brittleness of the concrete (Fig.2). The current study, diagonal cracking shear prediction is based on the above mentioned behavior.

2.4 Recommended Concrete Strength Regions

When discussing diagonal cracking shear strengths, there is a need for well-defined concrete strength regions. Based on the above discussion, previous studies [2, 3, 8, 14-22] and aggregate strength variations (Table 1) [5, 10, 12, 13], it is suggested that concrete can be categorized as NSC, optimal-strength concrete (OSC), and HSC. These can be defined as follows (Fig.2).

(1) Normal Strength Concrete (NSC)

The concrete compressive strength is less than that of optimum concrete strength. That is, in this strength region, concrete *DN* is lower than that of the aggregate. The recommended concrete compressive strength for NSC is less than or equal to 60 MPa. This value was estimated using the minimum tensile strength of aggregate (4.6 MPa, Table 1) and $f_t = 0.3 f'_c{}^{2/3}$, where f_t is splitting tensile strength of concrete [23]. The following equations are recommended for the prediction of modulus of elasticity of concrete E_c and f_t, $E_c = 4.73 f'_c{}^{0.5}$ and $f_t = 0.3 f'_c{}^{2/3}$ [24].

(2) Optimal Strength Concrete (OSC)

In this concrete region, the *DN* of concrete equals that of the aggregate. Considering past studies [10, 21], the recommended concrete compressive strength region is between 60 MPa and 100 MPa. In this region, the E_c and f_t are approximately related to f'_c by the expressions $E_c = 3.65 f'_c{}^{0.5}$ and $f_t = 0.59 f'_c{}^{0.5}$ [9, 23, 25, 26].

(3) High Strength Concrete (HSC)

Concrete strength is higher than the optimum concrete strength. That is, in this strength region, the concrete *DN* is higher than that of the aggregate. The recommended concrete compressive strength region is greater than or equal to 100 MPa. The following equations are recommended for the prediction of E_c and f_t, $E_c = 3.27 f'_c{}^{0.5}$ and $f_t = 0.51 f'_c{}^{0.5}$ [9, 26].

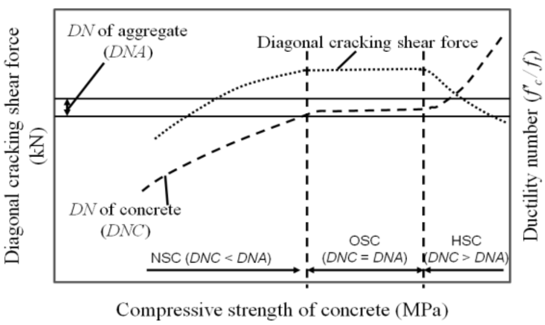


Fig.2 Effect of compressive strength of concrete on *DN* and diagonal cracking shear force of RC members
(NSC: Normal strength concrete,
OSC: Optimal strength concrete,
HSC: High-strength concrete)

3. DEVELOPMENT OF THE DIAGONAL CRACKING SHEAR STRENGTH PREDICTING EQUATION

3.1 Modeling Approach

According to past studies [2, 8], the principal factors resisting the development of shear crack are (1) the tensile strength of the concrete, (2) the interlocking action of aggregates across flexural cracks, (3) dowel action of longitudinal steel bars, and (4) shear stress of concrete in the compression zone. It is believed that the shear strength of reinforced concrete (RC) beams depends on the tensile strength of concrete, which in turn is related to its compressive strength. The dowel action of longitudinal steel bars and the shear resistance of concrete in the compression zone are considered to be defined by the longitudinal reinforcement ratio (ρ) and the compressive strength of concrete [2, 8, 27]. Therefore, the goal of this paper is to propose the following model to predict the diagonal cracking shear strength of slender RC beams with a shear span to depth (*a/d*) ratio greater than 2.5 without web reinforcement.

$$v_{cr} = k(geometry, reinforcement, load)\left(f'_c\right)^{\alpha} \quad (1)$$

where v_{cr} is diagonal cracking shear stress; k is a product of, the geometry of the beam; its longitudinal reinforcement, and its applied load; f'_c is compressive strength of concrete; and, α is a constant. For example, the ACI Code equation [28]

$$v_c = 0.167\left(f'_c\right)^{0.5} \quad (2)$$

fits this format, with k = 0.167 and α = 0.5.

3.2 Simplified Shear Strength Model

When the principal tensile stress at the neutral axis reaches the tensile strength of concrete, diagonal shear failure of the beam occurs. Mathematically, $V_{cr} = (2/3) f_t (c_1/d) b_w d$, can be used to calculate shear strength [27], where V_{cr} is the shear force at diagonal cracking, c_1 is effective shear depth, b_w is width of the beam, and d is the effective depth (Fig.3). The diagonal cracking shear stress is considered parabolic over the effective depth (Fig.4) with the maximum value at the neutral axis and it ($v_{cr} = Vcrbwd$ is given by

$$v_{cr} = \frac{2}{3}\frac{c_1}{d} f_t \quad (3)$$

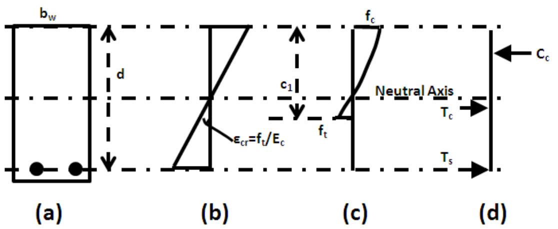


Fig.3 Strain compatibility and force equilibrium at section: (a) cross-section; (b) strain distribution, (c) stress distribution, (d) force distribution (where, ε_{cr} is cracking strain in concrete, C_c is compressive force due to concrete, T_c is tensile force due to concrete, and T_s is tensile force due to main steel)

Equation (3) shows that the v_{cr} depends on the tensile strength of concrete and the effective shear depth c_1 at the section under consideration. The tensile strength of concrete is a property of the material and generally related to its f'_c. On the other hand, the effective shear depth c_1 depends on the factored bending moment M_u and factored axial load at the section. Therefore, the value of c_1 depends on the critical shear span to depth ratio $M_u/V_{cr}d$, the f'_c, and the longitudinal reinforcement ratio ρ (Fig.3) [3, 27]. The value of $M_u/V_{cr}d$ can be taken as (*a/d*-1) for simply supported beams with point loading, where *a/d* is shear span to depth ratio [27]. For simply supported beams, the critical section for point loading is closer, approximately one beam depth away, to the location of maximum moment. This is because of existence of higher M_u at the section with the same V_u compared with other sections within the shear span [27, 29]. If the design requirements for beams under point loading are extended to beams under uniformly distributed loads, conservative prediction can be made [29].

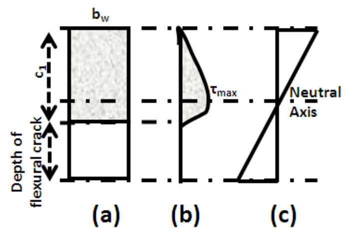


Fig.4 Shear stress distribution: (a) cross-section, (b) shear stress distribution, and (c) longitudinal strain distribution (where, τ_{max} is the maximum shear stress in effective shear depth)

If the shear strength of concrete, v_{cr}, is taken as $k_n f'_c{}^{1/2}$, then Eq. (3) can be reduced to

$$k_n = \frac{2}{3}\frac{c_1 f_t}{d} f'_c{}^{-1/2} \qquad (4)$$

A parametric study was undertaken to identify the influence of four parameters on the diagonal cracking shear strength of RC members computed using the proposed procedure and to define factor k and exponent α in Eq. (1) more precisely. The considered variables were a/d (ranging from 2.5 to 9.05), main longitudinal reinforcement ratio ρ (ranging from 0.47 to 3.39%), effective depth d (ranging from 132 to 1097 mm), and compressive strength of concrete f'_c (10.5 MPa to 194 MPa).

3.3 Empirical Expressions for Shear Strength

Five empirical expressions were considered in this study for comparison purposes. These equations are empirical in nature and have been based on numerous experimental data. A rational method for shear design is expected to compare well with the previous studies.

JSCE Code [30]

$$v_{cr} = 0.2 f'_c{}^{1/3}(d)^{-1/4}(100\rho)^{1/3}, f'_c \leq 80 \text{ MPa} \qquad (5)$$

ACI Code [28]

$$v_{cr} = 0.158 f'_c{}^{1/2} + 17.2\rho\frac{V_u d}{M_u}, f'_c \leq 70 \text{ MPa} \qquad (6)$$

Equation proposed by Khuntia and Stojadinovic [27]

$$v_{cr} = 0.537\sqrt[3]{\rho\left(f'_c\frac{V_c d}{M_u}\right)^{0.5}} \qquad (7)$$

This equation was found to accurately predict shear strength in concrete with a compressive strength between 28 MPa to 83 MPa.

Equation proposed by Fujita et al. [19]

$$v_c = 180 f'_c{}^{-1/2} d^{-1/2}(100\rho)^{1/3}\left(0.75 + 1.4/(a/d)\right)$$
$$80 \text{ MPa} \leq f'_c \leq 125 \text{ MPa} \qquad (8)$$

Equation proposed by Suzuki et al. [21]

$$v_c = 0.66(d)^{-2/5}(100\rho)^{1/3}\left(0.75 + 1.4/(a/d)\right)$$
$$60 \text{ MPa} \leq f'_c \leq 130 \text{ MPa} \qquad (9)$$

3.4 Results

The effect of each parameter was studied by varying its magnitude while maintaining the other variables. For each case, the corresponding k_n-value was computed using Eq. (3) and (4).

(1) Influence of shear span-depth ratio on shear strength

In any section of the beam, the depth of flexural crack is expected to increase with increasing bending moment. Thus, the cracking shear strength V_{cr} would decrease with increasing bending moment. Failure would occur at the section where V_{cr} equals the ultimate shear force (V_u). Based on this parametric study, the shear strength was found to be approximately proportional to $(a/d - 1)^{-1/6}$ for all ranges of concrete strength, beam depths, and reinforcement ratios. The variation of shear strength for a beam with f'_c = 124 MPa, d = 500 mm, and ρ = 1.59% [22] can be expressed as follows

$$k_{a/d} = 0.1\,(a/d - 1)^{-1/6} \qquad (10)$$

For deep beams, Eq. (10) gave conservative results (Fig.5).

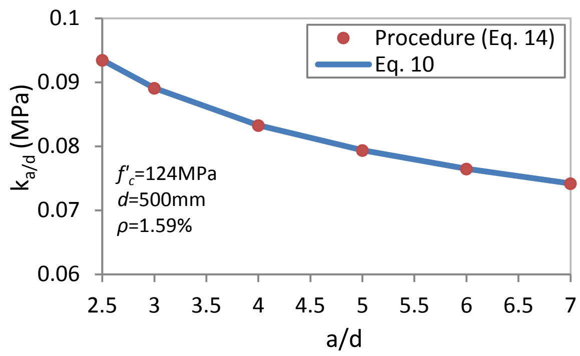


Fig.5 Influence of a/d on shear strength

(2) Influence of longitudinal reinforcement ratio on shear strength

When more tensile reinforcement is provided, the effective depth c_1 is higher in order to maintain axial force equilibrium, thereby increasing the shear strength of the beam (Eq. (3)). Shear strength was proportional to $(100\,\rho)^{1/3}$ for all ranges of concrete strength, beam depths, and a/d ratios considered in this study. Based on this parametric study, the variation of shear strength of HSC beams with an a/d ratio of 4, effective depth of 250 mm, and a concrete strength of 138 MPa [2] can be expressed as

$$k_\rho = 0.096\,(100\,\rho)^{1/3} \qquad (11)$$

(3) Influence of effective depth of the beam

When the size of a beam is increased, the location of the critical section of a beam would be closer to the load point, giving a greater moment region [29]. Therefore, the effective shear depth c_1 of RC beams decreases with increasing beam depth. Based on this parametric study, the shear strength was found to be proportional to $d^{-1/4}$ for all ranges of a/d, ρ, and f'_c. The variation of shear strength for beams with a/d = 4, f'_c = 183 MPa and ρ = 3.04% [2] can be expressed as follows

$$k_d = 0.078\,(d)^{-1/4} \qquad (12)$$

(4) Influence of compressive strength of concrete on shear strength

When HSC or OSC is used in place of NSC, the neutral axis depth decreases to maintain force equilibrium. Thus, the shear strength factor k_n is expected to be lower. It has been found that the shear strength factor k_{NSC}, k_{OSC}, and k_{HSC} were proportional to $f'_c{}^{-1/3}$, $f'_c{}^{-1/2}$, and $f'_c{}^{-2/3}$ for all ranges of reinforcement ratios and *a/d* ratios considered in this study, respectively. A simplified expression is suggested by fitting the model result for a beam with 1.53% reinforcement ratio, effective depth of 250 mm, and *a/d* ratio of 3 [19] as follows

$$v_{cr} = k_{NSC} f'^{1/2}_c = 0.5 f'^{1/6}_c, \text{ for NSC} \quad (13a)$$
$$v_{cr} = k_{OSC} f'^{1/2}_c = 1.37 f'^{0}_c, \text{ for OSC} \quad (13b)$$
$$v_{cr} = k_{HSC} f'^{1/2}_c = 2.1 f'^{-1/6}_c, \text{ for HSC} \quad (13c)$$

3.5 Proposed shear strength equation for RC beams without transverse reinforcement

Based on the previous parametric study, considering the influence of primary parameters *a/d*, *ρ*, *f'*$_c$, and *d* only, the shear strength of RC beams without web reinforcement can be expressed as

$$v_{cr} = f_v\,(100\rho)^{1/3} d^{-1/4} (a/d - 1)^{-1/6} \quad (14)$$

where,

$f_v = 0.49 f'^{1/6}_c$, for $f'_c \leq 60$MPa
$f_v = 0.97$, for 60 MPa $< f'_c <$ 100 MPa
$f_v = 2.09 f'^{-1/6}_c$, for $f'_c \geq 100$ MPa

4. VERIFICATION USING EXPERIMENTAL DATA

To check the validity of proposed parametric Eq. (14), 216 test beams from nine different investigators was examined (see Table 2). The results were also compared with those calculated using other empirical expressions.

Table 2 Outline of data used; range of parameters

Concrete type [References]	Number	f'_c (MPa)	ρ (%)	d (mm)	a/d
NSC [14-21]	111	10.5-51.9	0.47-3.04	132-1097	2.5-9.05
OSC [18, 19, 21]	30	68.8-98.8	0.5-1.53	150-1000	2.5-4.0
HSC [19-22]	75	101-193.8	0.54-3.39	150-1000	2.5-4.0

The results presented in Table 3 show that the proposed parametrically derived Eq. (14) fits experimental data and are statistically similar (Fig.6). Among other methods, that suggested by Suzuki et al. [21] was found to be reliable for HSC design purposes wheras the predictions by Fujita et al. [19] were found to be conservative for HSC. However, the shear of HSC beams (f'_c > 100 MPa) is affected by the strength anisotropy of individual rock. Therefore, the shear strength of concrete scatters around 100 MPa- 150 MPa and a conservative design approach is recommended for HSC beam designs. On the other hand, the predictions provided by the JSCE [30] method and ACI [28] equation were found to overestimate the shear strength of HSC beams. However, the JSCE [30] method was most reliable for design purposes than the ACI code [28].

Table 3 Comparison of test results with predicted results by different investigators

Concrete type	Test / predicted ratio, mean (standard deviation)					
	Eq. (5) by JSCE	Eq. (6) by ACI	Eq. (7) by [27]	Eq. (8) by [19]	Eq. (9) by [21]	Proposed Eq. (14)
NSC	*1.19 (0.18)	*1.19 (0.28)	*1.20 (0.22)	0.50 (0.17)	0.84 (0.15)	1.00 (0.13)
OSC	0.97 (0.15)	0.72 (0.19)	1.07 (0.25)	0.94 (0.11)	*0.94 (0.09)	1.00 (0.13)
HSC	0.99 (0.18)	0.79 (0.18)	1.17 (0.24)	1.20 (0.18)	1.04 (0.15)	1.16 (0.17)
All	1.09 (0.21)	0.99 (0.31)	1.17 (0.24)	0.80 (0.37)	0.92 (0.17)	1.05 (0.16)

*: predictions inside recommended concrete strength

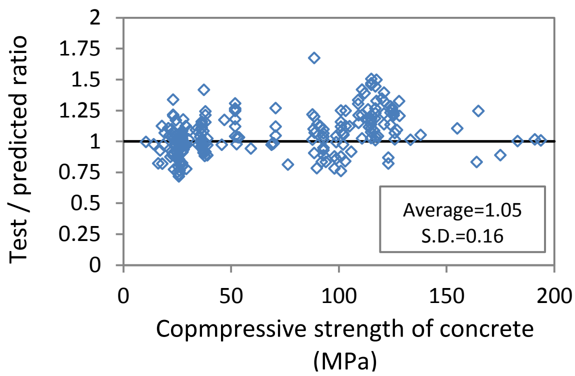


Fig.6 Comparison of experimental results with proposed equation: influence of compressive strength of concrete.

5. CONCLUSIONS

Based on a simple model, parametric study, and comparison with numerous experimental results, different code provisions, and empirical expressions, the following conclusions can be drawn:

(1) The proposed shear strength prediction equation is suggested for the design of slender RC beams with a concrete strength between 10.5 MPa to 193.8 MPa.
(2) The present JSCE method and ACI equations for evaluating the shear strength of HSC beams (outside recommended concrete strength) need to be modified as suggested in this paper.

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