

# SHEAR CRACKING BEHAVIOR OF HIGH-STRENGTH PRESTRESSED REINFORCED CONCRETE BEAMS

Sudhira DE SILVA<sup>\*1</sup>, Hiroshi MUTSUYOSHI<sup>\*2</sup>, Shingo ASAMOTO<sup>\*3</sup>, and Mayuko TAKAGI<sup>\*4</sup>

## ABSTRACT

An experimental program was conducted to investigate the shear cracking behavior of both normal strength concrete and high strength concrete I-shaped beams. All beams were tested by focusing on the influence of compressive stress in concrete due to prestress and compressive strength of the concrete on the shear crack width. It was found that there is a linear relationship between shear crack width and stirrup strain in the both normal strength and high strength concrete beams. Shear crack widths are smaller in PRC beams.

**Keywords:** high-strength concrete, prestressing, shear crack width, stirrup strain

## 1. INTRODUCTION

Cracking in concrete structures has received enormous research attention, as durability and serviceability of concrete structures are often dependent on crack formation. Excessive crack widths can impair corrosion resistance of structures exposed to the severe environment. Therefore, control of cracking is more important for serviceability. Recently, the use of small and economic reinforced concrete sections has increased because of their simplicity [1]. The high strength concrete (HSC) has been widely used in the construction field because the increased strength associated with HSC provides a better solution to reduce size and weight of concrete structural elements. Increased mechanical strength and improved various characteristics such as improved durability, low permeability, low creep and better workability encourage design engineers to use HSC [1, 2]. In addition, the HSC permits a facility to accelerate construction.

Prestressed concrete beams incorporating with non-prestressed steel reinforcements are built today with an allowance of tension in concrete, which are well known as partially prestressed concrete (PPC) or prestressed reinforced concrete (PRC) in Japan. Cracking behavior of reinforced concrete (RC) beams and PRC beams have been investigated over the last five decades by

conducting comprehensive experiments [3-6]. A previous experimental study conducted by the authors [4] has investigated effects of prestressing force, stirrup ratio, and side concrete cover on shear crack width of RC and PRC beams. More studies related to HSC are concerned on concrete strength range from 60 MPa to 80 MPa, but very few studies have been done with concrete strength of over 100 MPa. It has been reported that the crack plane in HSC is relatively smooth compared to that in normal strength concrete (NSC) as crack passes through aggregate in HSC whereas crack goes around the aggregate in NSC [7]. Such phenomenon results to reduce contribution from shear force due to aggregate interlocking action ( $V_a$ ) in a diagonally cracked HSC beam. Therefore to compensate the reduction in  $V_a$ , a higher shear due to stirrup stresses ( $V_s$ ) may be needed. Moreover, due to the higher tensile strength of high strength concrete, a higher cracking shear is expected. Significant effects of concrete strength on shear capacity and properties of structural elements found in previous studies [4, 5] imply that concrete strength can have influence on shear crack width, but there is no known study of the effect of concrete strength on shear crack width. The objective of this study was to investigate the effect of concrete strength on shear cracking behavior of RC and PRC beams experimentally.

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\*1 Graduate School of Science and Engineering, Saitama University, PhD Student, Member of JCI

\*2 Graduate School of Science and Engineering, Saitama University, Professor, Member of JCI

\*3 Graduate School of Science and Engineering, Saitama University, Asst. Professor, Member of JCI

\*4 Graduate School of Science and Engineering, Saitama University, MEng Student, Member of JCI

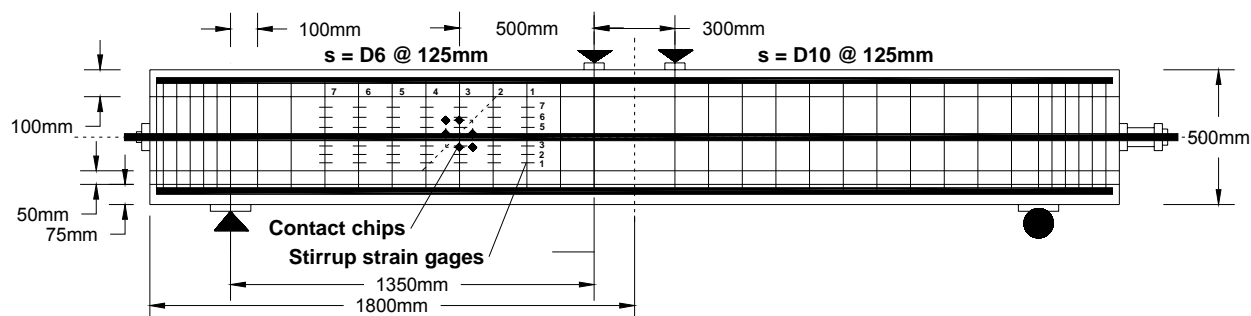


Fig. 1 Layout of the specimens

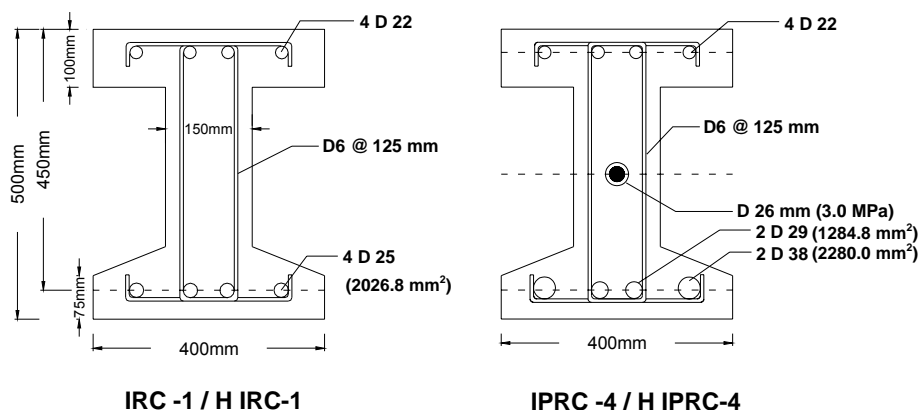


Fig. 2 Cross sectional details

Table 1 Experimental variables

Beam #	Non-prestressed reinforcement		Prestressing force (kN)	Concrete
	Top	Bottom		
IRC-1				NSC
H IRC-1	4 D22 ( $d_s=40$ mm)	4 D25 ( $d_s=450$ mm)	-	HSC
IPRC-4				NSC
H IPRC-4		2 D29 and 2 D38 ( $d_s=450$ mm)	375.0 kN (3.0 MPa)	HSC

Table 2 Mechanical properties of reinforcements

Type of bar	$\Phi$ (mm)	Type	$f_y$ (MPa)	$E_s \times 10^3$ (MPa)
Deformed bar	D6	SD 345	438	200
	D10		376	
	D22		397	
	D25	USD 685	720	206
	D29		735	
	D38		726	
PC bar	26	SBPR 1080/1230	1205	200

## 2. EXPERIMENTAL PROGRAM

In order to investigate the influence of compressive strength of concrete including prestressing force on shear crack width in PRC beams, the following experimental program was carried out. The compressive strength of concrete was 40 MPa for normal strength concrete (NSC) specimens and that was 100 MPa for high strength concrete (HSC) specimens at the age of 28 days under moist curing condition. Test specimens consisted of two RC and two PRC beams having I-shaped cross section. A total length of beam is 3600 mm. The typical layout and cross sectional details of the specimens are shown in Figs. 1 and 2, respectively. The experimental variables are

summarized in Table 1. The mechanical properties of reinforcements used in the experiment are listed in Table 2. Table 3 shows concrete properties used. In all test specimens, stirrups were provided with two different sizes of stirrup bars; D6 bars were in the left span of the beams and D10 bars were in the right span of the beams. This arrangement of different sizes of bars was necessary so as to ensure that the main shear crack would occur in the left span of the beam. The four point symmetrical loading with a distance of 300 mm between the loading points ( $a/d$  of 3.0) was statistically applied to all specimens. Contact chips were pasted on the concrete surface at 500 mm from the left-loading point of the beam in the shear span region so as to

measure the principal strains and their directions. A digital microscope, which has a precision of 0.001 mm, was used to capture digital photographs of the crack occurred in the left span of the beam (Fig. 3). The digital image captured at the crack location was used to measure the crack width. At three arbitrary extracted points, perpendicular to the crack surface at close to the location of necessity was measured. The average of measured three crack widths was used as the crack width at that location (Fig. 3b).

Table 3 Concrete properties

	NSC	HSC
Average compressive strength of concrete (MPa)	41.7	100.4
Slump / flowability* (mm)	174.0	560.0*
V- funnel test (sec)	-	15.62
water / cement ratio	0.47	0.23
Young's modulus (GPa)	-	38.4

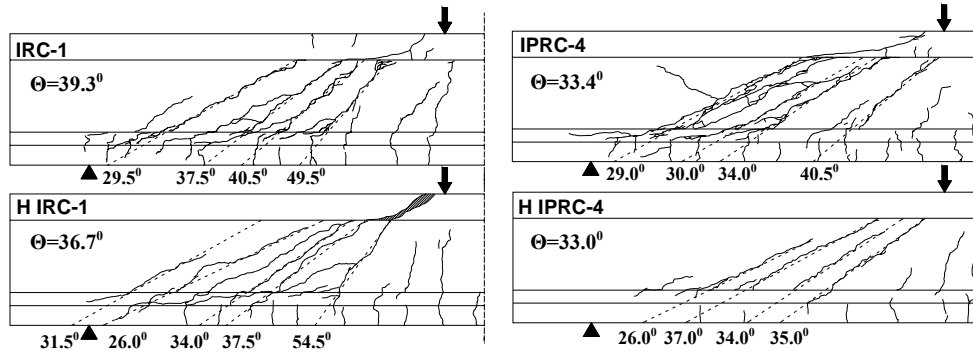


Fig. 4 Failure crack patterns and crack angles

### 3. TEST RESULTS AND DISCUSSION

#### 3.1 Failure Mode and Crack Pattern

Crack patterns at failure for all specimens are shown in Fig. 4. All specimens failed in shear failure mode (Table 4) with wide shear cracks in the shear span region. The inclination of shear cracks was determined by averaging crack angle measured in the shear span region. It can be seen that in PRC beams the inclination of shear cracks slightly decrease due to prestress. The effect of strength of concrete on shear crack angle was insignificant. However, the number of shear cracks appeared in HSC specimen was greater than that in NSC specimen. The spacing in between shear cracks was smaller in HSC specimens. Specific discussion about shear crack spacing is given in section 3.4.

#### 3.2 Effect of Compressive Strength of Concrete

In the case of HSC it has been reported [7, 8] that large autogenous shrinkage can occur and

Table 4 Failure load

Beam #	Failure load (kN)	$f'_c$ (MPa)	Failure mode
IRC-1	559.7	40.3	Shear failure without yielding tension reinforcements. All stirrups yield.
H IRC-1	749.5	100.4	
IPRC-4	745.0	43.2	
H IPRC-4	1032.3	100.4	

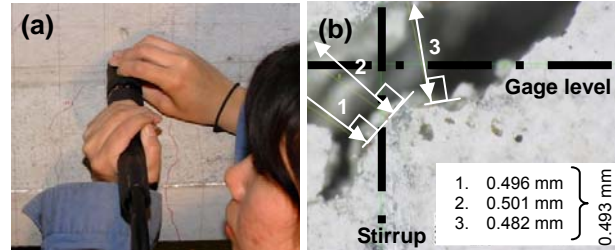


Fig. 3 (a) Measuring crack width by using digital microscope, (b) Digital image of measured crack width

shrinkage cracking can be appeared due to insufficient curing. This study was carried out with proper moist curing. As a result, there were no shrinkage cracks appearing in HSC specimens.

Fig. 5 shows the relationship between shear force and maximum stirrup strain. It can be seen that a similar rate of increment in stirrup strain with increasing shear force in both NSC and HSC specimens. Relation between shear force and maximum shear crack width is presented in Fig. 6. At a particular shear force, maximum shear crack width is almost similar in RC beams (Fig. 6a) while it is greater in PRC beams (Fig. 6b) compared with corresponded HSC beams. This is attributed to higher improvement in bond between steel and concrete in HIPRC-4 beam which was cast with larger diameter bars. The higher shear cracking load was observed due to higher tensile strength of HSC specimens. After shear cracks appeared, the increment rate in shear crack width with increasing shear force is similar in both normal strength specimens and high strength concrete specimens.

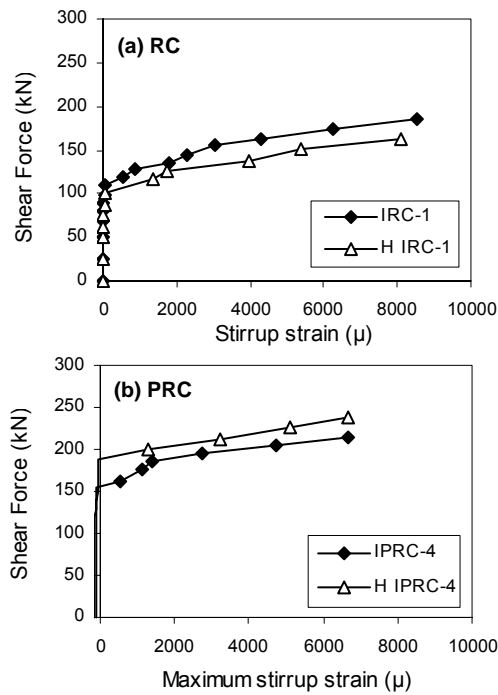


Fig. 5 Load vs. Maximum stirrup strain

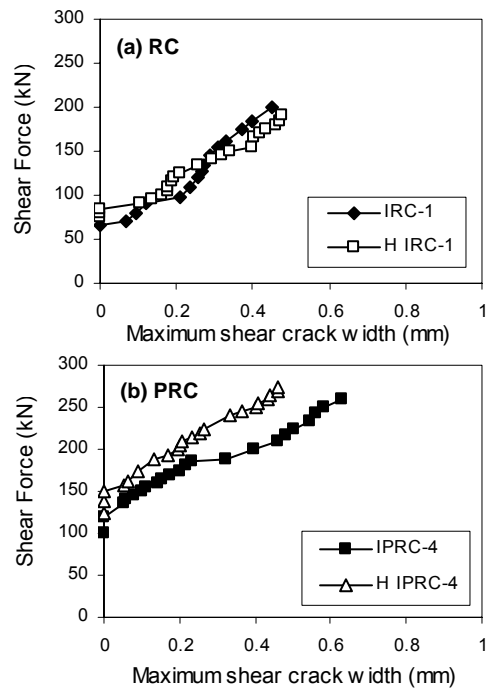


Fig. 6 Load vs. Maximum shear crack width

### 3.3 Shear Crack Width and Stirrup Strain Relationship

Fig. 7 shows the relationship between shear crack width and stirrup strain. Fitting curves obtained from a linear regression analysis represent the relationship between the average values of shear crack width and stirrup strain. For both HSC and NSC beams, the linear variation between the shear crack width and the stirrup strain determined from the regression analysis shows a reasonable representation of measured data. The average shear crack width variation with

increment of stirrup strain is almost same in the both NSC and HSC of RC and PRC specimens. This is due to small variation of tensile strength of concrete in NSC and HSC specimens. For RC beams, the shear crack width data obtained for NSC specimen are more scattered comparing with the data obtained for HSC specimen. For HSC specimens, some of cracks may penetrate through aggregates due to strong bond between aggregate and hardened paste, while that of NSC specimen, the cracks always occur in the interface between aggregates and hardened paste. Therefore observed crack plane in NSC specimens was not smooth. The axial compressive force from prestressing (375 kN) in PRC specimens compared to RC specimens caused further reduction in scattering of shear crack width values.

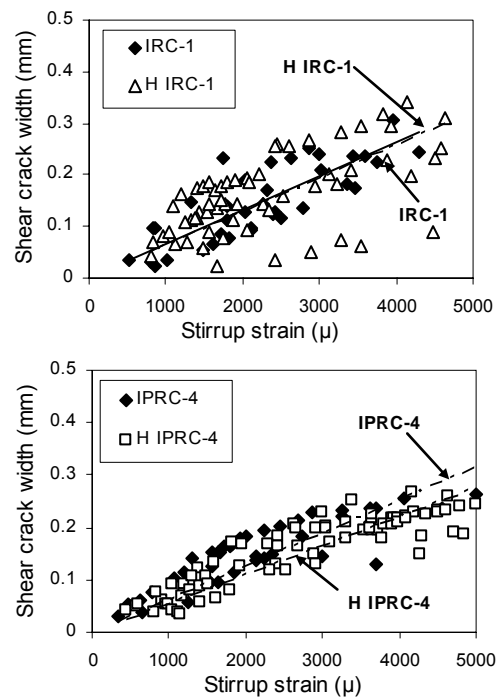


Fig. 7 Shear crack width and stirrup strain relationship

Table 5 Average shear crack spacing

Beam #	Horizontal crack spacing, $S_{mx}$ (mm)	Vertical crack spacing, $S_{mv}$ (mm)
IRC-1	167.00	108.29
H IRC-1	187.00	79.00
IPRC-4	287.15	157.30
H IPRC-4	119.12	68.37

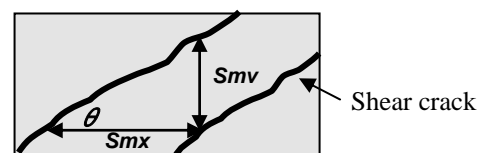


Fig. 8 Interpretation of shear crack spacing

### 3.4 Shear Crack Spacing

Table 5 shows the average shear crack spacing  $Smx$  and  $Smv$  (as shown in Fig. 8) for NSC and HSC specimens. The HIPRC-4 specimen, which has a large amount of reinforcement steel shows comparatively smaller shear crack spacing than the others. This difference is attributed to improvement of bond effect due to large amount of reinforcing steel in HSC specimen. However, such significant difference could not be seen in other specimen. The smaller shear crack spacing is influenced to smaller crack width in HIPRC-4 specimen compared to IPRC-4 specimen at same shear force level (Fig. 6b).

## 4. NUMERICAL SIMULATION

### 4.1 Numerical Method

The numerical simulation was carried out using Response-2000 numerical model, which was developed based on the Modified Compression Field Theory (MCFT). Experimental variables and actual compressive strength of concrete of the test specimens were used in the numerical model. The numerical simulation in the model combines a plane section analysis for flexure with the modified compression field theory for shear that accounts for strain compatibility and uses the tensile and compressive stress-strain relationships for diagonally cracked concrete [9]. In this method, the spacing of shear crack was accounted so as to determine shear cracking load and ultimate load. The MCFT that reduces the shear stress was carried for the concrete when large number of shear cracks appeared in the section. The crack spacing is a function of the crack control characteristics of the longitudinal and transverse reinforcement described in the crack spacing model of the numerical program. The sectional analysis was performed at a section located at a distance of 500 mm (approximately at a distance “ $d$ ” and at the section along stirrup location) from the face of the loading point in the shear span. The moment-to-shear force ratio is 0.9 at the selected section. Although the ACI 318-02 code limits the yield stress of shear reinforcement to 400 MPa, the experimentally measured value of yield stress 438 MPa was used in simulation of the model presented here. In addition, constitutional models for concrete and other material were used as described in the numerical program. The material properties of concrete and steel used in the numerical model were based on the experimentally measured values and details provided by manufactures (see Tables 2 and 3).

### 4.2 Comparison of Load – Mid Span Deflection Relationship

Fig. 9 shows the load mid span deflection relationship for HIRC-1 and HIPRC-4 specimens. The prediction of the relationship using numerical program generally showed good agreement with experimental results except at the ultimate failure load. One of the contributing factors for the lower value predicted by the model at the ultimate load can be the size effect as explained in preceding section. The failure of concrete section occurred due to large local stresses induced at the crack. The prediction of flexural cracking load, shear cracking load and load at stirrup yielding agreed well with experimental results. In addition, moment-shear interaction diagram was used to predict the failure mode as explained in the numerical program [9].

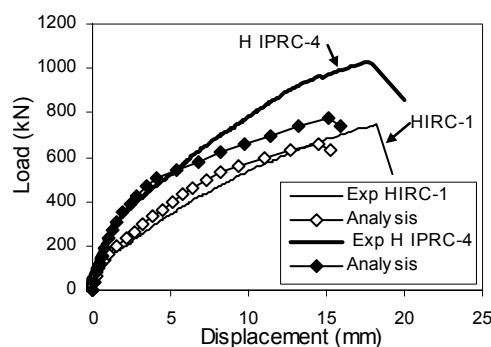


Fig. 9 Load-mid span deflection relationship

### 4.3 Crack Spacing Model

The shear crack width predicted from this numerical program equals to the product of the average shear crack spacing and principal tensile strain. The crack spacing model based on the CEB-FIP Model code 1978 [8] is:

$$Sm\theta = 2c + 0.1 \frac{d_b}{\rho} \quad (1)$$

where; “ $c$ ” is the diagonal distance to the nearest reinforcement in the section considered and “ $d_b$ ” is the diameter of the nearest bar and “ $\rho$ ” is the percentage of steel. The spacing of the shear cracks depend on the crack control characteristics of both the longitudinal reinforcement and the transverse reinforcement.

### 4.4 Comparison of Shear Crack Width Values

Fig. 10 compares the relationship between shear force and shear crack width obtained from experimental data and model response predicted from the numerical program Response-2000. The shear crack width values predicted from the numerical program show good correlation with

experimental results. In addition numerical program estimated the flexural cracking load, and the shear cracking load very accurately.

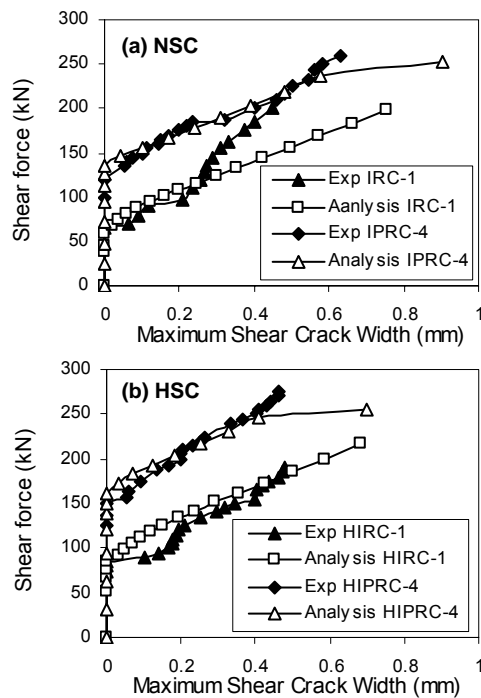


Fig. 10 Comparison of numerical results for maximum shear crack width

#### 4. CONCLUSIONS

The study illustrated an experimental investigation of effect of concrete strength and prestressing force on the shear crack width of I-shaped RC and PRC beams under static loading. In addition, numerical simulation was carried out based on Modified Compression Filed Theory (MCFT). The following conclusions are derived from this study:

- (1) The high strength concrete specimens show larger shear cracking load in RC and PRC specimens compared to normal strength concrete specimens.
- (2) After shear cracks appeared, the increment rate in stirrup strain and maximum shear crack width with shear force is similar in both NSC and HSC specimens. At a particular shear force, maximum shear crack width is almost similar in RC beams while it is greater in PRC beams compared with corresponding HSC beams. That is, prestressed reinforced concrete beams made with HSC show smaller shear crack width compared to corresponding NSC beams.

- (3) It was found that the shear crack width variation with stirrup strain in a linear manner in both NSC and HSC specimens.
- (4) The Modified Compression Field Theory based numerical simulation was showed good correlation with experimental results.

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