THE STANDARDIZATION OF TENSION SOFTENING CURVES OBTAINED FROM THE UNIAXIAL TENSION TESTS OF CONCRETE

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ABSTRACT

The paper is dedicated to proposal of standard tension softening curves by some formulas with comparison to experimental data. Knowledge of the tension softening process of concrete is essential to understand fracture mechanism, and next to analyze fracture behaviour, and further to evaluate properties of concrete. For the last eight years, many different tests on uniaxial tension with elimination of secondary flexure have been performed in Tohoku Institute of Technology. Based on such results some new equations were proposed describing tension softening curves.

Keywords: tension softening curve, uniaxial tension, secondary flexure, tensile strength

1. INTRODUCTION

The application of fracture mechanics for the analysis of the concrete structures requires understanding the behaviour of concrete under tensile loading. The best way to obtain tension softening curve is applying tension directly to a concrete specimen, because it is possible then to obtain immediately both the tensile strength and the tension softening curves from specimens loaded in a possibly uniform way. The situation does not occur in other experiments dedicated to tension, such as splitting tests or three or four points bending test in which stress-gradient exists.

In order to evaluate properties of concrete, it is necessary to understand fracture mechanism and fracture behaviour, so an analysis of tension softening process of concrete is essential. To obtain reliable results, the number of problems must be resolved to prevent or to minimize the secondary flexure. The flexure may impair the resulting data, especially the load-deformation curve, and affects also various fracture parameters that are to be evaluated. Many researchers ignore the secondary flexure, [1-2]. Problem was solved by Carpinteri et al. in 1994, [3] and next by Akita et al. in 2000, when a specially designed adjusting gear system has been developed to eliminate the secondary flexure, [4-6].

The reduction of the observed peak load or tensile strength due to the secondary flexure is significant. The reduction easily exceeds 10 % and sometimes exceeds even 20 %, [7]. Therefore, it is necessary to prevent completely or to minimize the secondary flexure for obtaining reliable results. Series of the experiments with elimination of the secondary flexure in manual and automatic operation of adjusting gear system were done, but the results with elimination of secondary flexure performed by automatically controlled gear system are only presented in this paper. Because, the automatic operation gives better results for both load-deformation curves and tension softening curves.

2. UNIAXIAL TENSION TESTS PROCEDURE

The uniaxial tension tests were performed using ordinary concrete, i.e. compressive strength is about 30 MPa and the maximum aggregate size is 20 mm. The specimen shape is the rectangular prism of dimensions $100 \times 100 \times 400$ mm. The notches introduced to prevent multiple cracks and called primary notches were cut on two side faces, perpendicular to the cast and bottom planes. Besides these, other notches called guide notches were made on the cast and bottom faces in order to prevent overlapping of cracks.

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The important aspect is stability of fracture which should be realized by selection of a proper load-control system, otherwise the test results will be incredible. It was a reason, why experiments were performed with the closed-loop loading machine that allows ensuring the stable fracture. The extensometers, aligned at the centre of the prisms, were attached on the all four side faces. The adjusting gear system, [4-5] combined with the universal joints as shown in Fig. 1 can eliminate the secondary flexure which can produce a significant error during uniaxial tension testing of concrete.

The elimination of the secondary flexure was realised by some procedure as follows. If one side of the specimen is more stretched than the opposite side, this side should be contracted. It is done by tightening of the adjusting gear fixed on this side, and activated until a proper balance in elongation is reached. For the operation, it is necessary to observe the deformations (elongation) on all four sides of the prism under loading. When a certain side is stretched and its opposite side is already contracted, the opposite side should be loosened as not to introduce unnecessary forces into the specimen.

Fig. 1 Adjusting gear system and experimental set up

3. PROBLEM OF SECONDARY FLEXURE

Problems encountered during investigating tension-softening behaviour under the uniaxial tensile load are unstable fracture, secondary flexure, multiple cracks, and overlapping cracks. Concrete is a heterogeneous material. It is possible to distinguish the aggregates and other parts by their properties. The local stress and strain gradients appear immediately when an external

load is applied. Thus, the perfect experiments should restrict such influence of disturbance caused the load eccentricity and/or a one side cracking; this problem was discussed by Hordijk, [8] and by Akita [5].

The effect of such secondary flexure can be seen in Fig. 2, where an example of a certain actual load-deformation curves (diagram P-δ) on four side faces is shown. In the case of the experiments without the elimination of secondary flexure, the load deformation curves are misleading because of a harmful influence of such flexure. For example, four curves are quite different and one side of the specimen was compressed.

In the case of elimination of secondary flexure, all sides of the specimens were elongated at the same magnitude and the shape of the curves is nearly similar, like in Fig. 3.
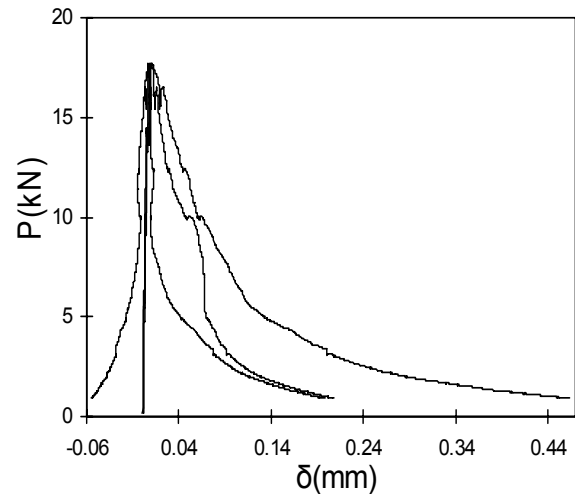

Fig. 2 An example of the load-deformation curves for four side faces, without elimination of the secondary flexure
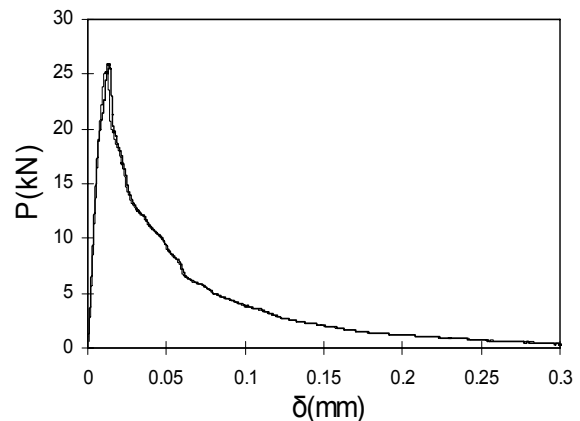

Fig. 3 An example of the load-deformation curves with elimination of the secondary flexure

During experiments of uniaxial tension tests with elimination of secondary flexure, the adjusting gear system introduces additional moment and makes the resultant force eccentric. However, an eccentric force is a serious problem only when it produces stress or strain gradient. In the case of these experiments, such the eccentric force is no problem, because it eliminates the secondary flexure and makes a uniform tensile stress distribution in the cross section of the specimen.

When tension softening curve is analysed, crack opening displacement (w) is evaluated by the following Equation(1):

$$w = \delta - \frac{PL}{EA} - \delta\,r \qquad (1)$$

where,
- δ : the observed deformation
- P : the applied load
- L : measuring length
- E : Young's modulus
- A : area of the initial cross section (of the ligament)
- δr : residual deformation, when the load from the maximum value decreases to zero.

It is assumed that the material outside the softening zone behaves elastically after passing the maximum load, i.e. when the specimen is on the softening branch of the curve. The process of evaluation of crack opening displacement was described in Fig.4, as shown by schematic load-deformation curve.
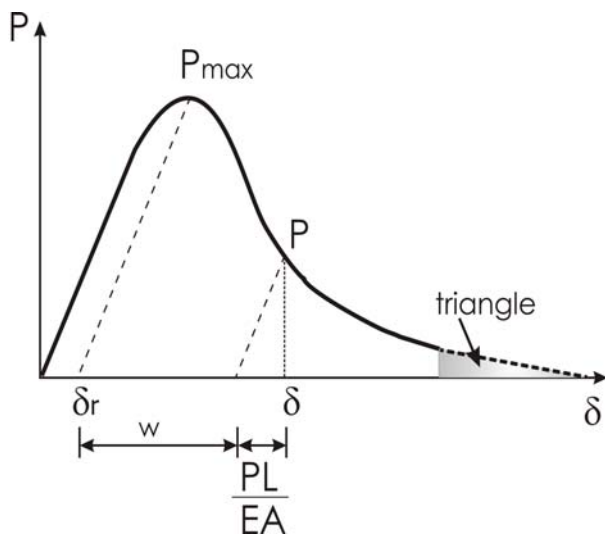

Fig. 4 Schematic explanation of the crack opening displacement

The feature triangle is obtained from extrapolation line of the experimental curve at the broken point. Tension softening curve is in the relationship between σ (cohesive stress: P/A) and crack opening displacement (w), and the fracture energy was evaluated as the area under the tension softening curve.

The Young's modulus is calculated from the slope of the P-δ curve in region between 10% and 65% of the maximum load in the ascending branch.

4. DESCRIBING OF TENSION SOFTENING CURVES

The investigations were aimed at analysing the existing equation describing tension softening curves in case of 15 experiments with elimination of secondary flexure of concrete by automatically controlled gear system, as shown in Fig.5.
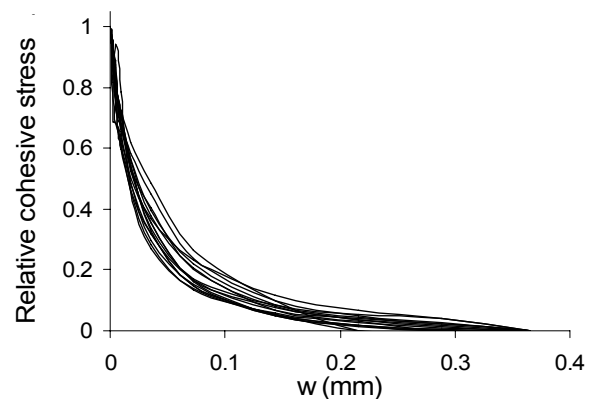

Fig. 5 Tension softening curves for 15 specimens

At the beginning, the existence equations describing tension softening curve were checked. The first was an algebraic and an exponential term, Equation(2), proposed by Reinhardt et al. (1986), [9] and the second expression was the power function proposed by Li (2002), [10] as Equation(3).

$$\sigma_r(w) = \left\{1 + (a \cdot w)^3\right\} \exp(-b \cdot w) - w \cdot (1 + a^3) \exp(-b) \qquad (2)$$

$$\sigma_r(w) = 1 - \exp\left[-\left(\frac{a}{w}\right)^b\right] \qquad (3)$$

where,
- w : crack opening displacement [mm]
- $\sigma_r(w)$: relative cohesive stress - stress per tensile strength

All parameters: a, b, c, d, e in Equations (2), (3), and later Equations (4), (5), (6), (7), were evaluated through the least square method and statistical parameters (R^2, chi-squared) by Excel.

It was occurred that only Equation(2) could predict the present values with good coefficient of correlation $R^2 = 0.997$, but it gives too small w_c, i.e. predicted critical crack opening displacement is around 0.23, as shown in Table 1. The average crack opening displacement for all experimental curves was 0.365, so it was excluded from further analysing, because it would not be able to describe exactly tension softening curves for analysed specimens. It should be noticed that critical opening displacement is an important parameter to describe a tension softening curve.

The equations proposed by Reinhardt, Equation(2) and Li, Equation(3) described tension softening curve appropriate for their experimental results, but occurred not enough for experiments with elimination of secondary flexure.

Table 1 Values of evaluated parameters and coefficients for the former formulas

	Equation(2)	Equation(3)
a	21.358	0.011
b	42.570	1.300
R^2	0.9970	0.9674
chi-squared	0.0088	0.0074
w_c [mm]	0.23	0.6

where,

w_c : critical crack opening displacement was evaluated form curves in the intersection of abscissa

The approximated results by Equations(2) and (3); are shown in Fig. 6. Open circles show the averages of the present 15 curves (original data).
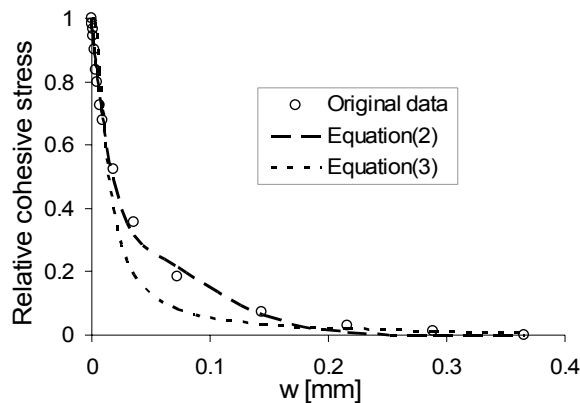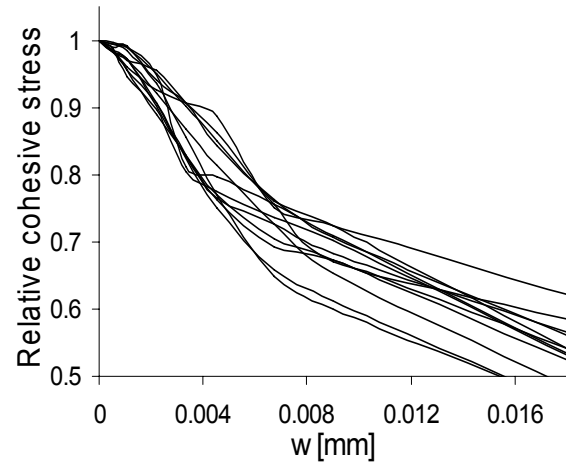

Fig. 6 Approximation data by former formulas

Fig. 7 Parabolic shape of the curves of relative cohesive stress

Moreover, it was noticed also that curves at the top were rounded like parabola, as shown in Fig. 7. So, it was undertaken an attempt on finding other expression as in following equations:

$$\sigma_r(w) = 1 - \frac{a \cdot w + b}{c \cdot w + d} + \frac{b}{d} \tag{4}$$

$$\sigma_r(w) = 1 - a \cdot \exp(b \cdot w) + c \cdot \exp(d \cdot w) + a - c \tag{5}$$

$$\sigma^r(w) = 1 - a \cdot \exp(-b \cdot w^2) + \frac{1}{c \cdot w + d} + a - \frac{1}{d} \tag{6}$$

$$\sigma_r(w) = 1 - a \cdot \exp(-b \cdot w) + \frac{1}{c \cdot w + d} + a - \frac{1}{d} \tag{7}$$

For all examined curves, parameters like coefficients of correlation, chi-squared parameters and critical crack opening displacement were evaluated, as given in Table 2.

Table 2 Values of parameters and coefficients

	Eq.(4)	Eq.(5)	Eq.(6)	Eq.(7)
a	10.114	114.642	22.156	-0.259
b	-0.0004	0.0054	-0.0027	15.378
c	9.575	0.804	45.318	84.884
d	0.203	-51.395	0.950	1.296
R^2	0.9989	0.9954	0.9989	0.9996
chi-squared	0.0026	0.0110	0.0025	0.0019
w_c [mm]	0.360	0.310	0.350	0.365

The shapes of approximated curves were shown in Fig. 8 and Fig. 9. All of them have very good coefficient of correlation and exactly describing average data (marked as circles) based on 15 specimens. The best one seems to be Equation(7), considering w_c, R^2 and ch-squared in Table 2.
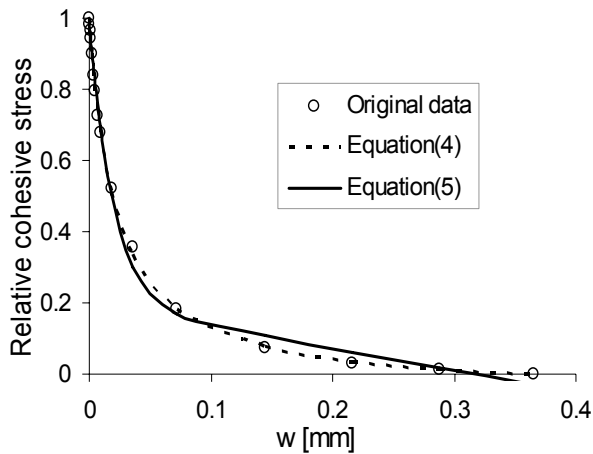

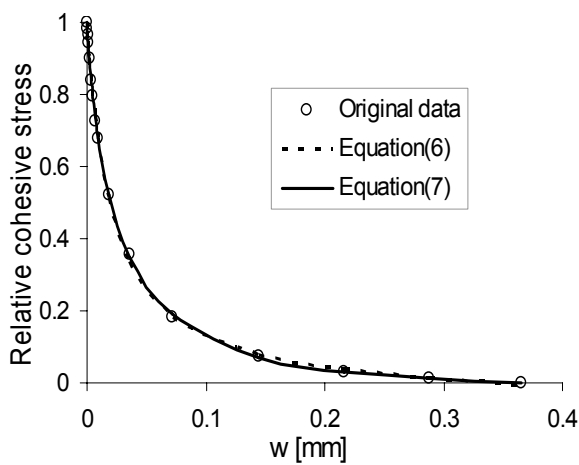Fig. 8 Approximation data by examined Equations (4) and (5)

Fig. 9 Approximation data by examined Equations (6) and (7)

The curves expressed by new formulas were in good agreement with original data concerning experiments with perfect elimination of secondary flexure of concrete. However, the deeper analysis of top of curves, shown in Fig. 10 and Fig. 11 let propose the Equation(7) as the best one to describe the original data. This equation has also the lowest chi-squared coefficient, like in Table 2.

In the near future an attempt will be made to find another way to describe tension softening curves based on the present data. Such possibilities are given with the approach of artificial intelligence methods, which were applied

with success in previous examinations concerning concrete engineering problems, [7, 11].
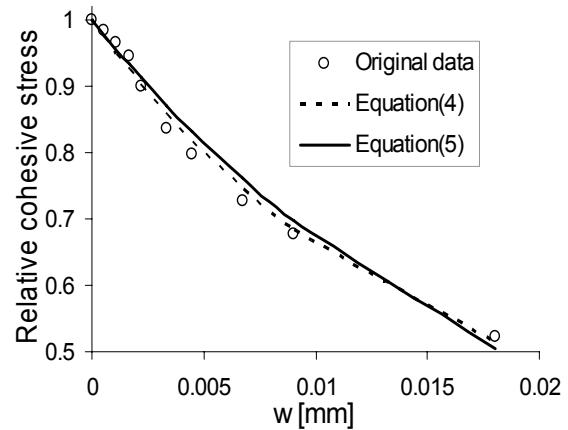

Fig. 10 Approximation of parabolic shape at the top of the curves for Equations (4) and (5) of relative cohesive stress
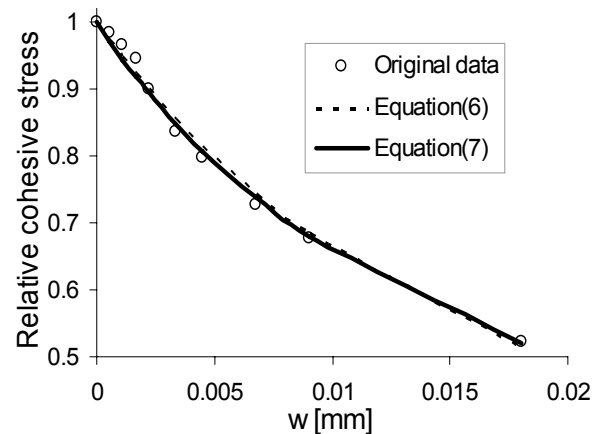

Fig. 11 Approximation of parabolic shape at the top of the curves for Equations (6) and (7) of relative cohesive stress

5. CONCLUSIONS

Description of tension softening curve by some new expressions has been examined. The special attention was paid for proper analysing the parabola shape at the top of curves to express the relationship between relative cohesive stress and crack opening displacement.

The following conclusions are drawn based on the present examinations.

(1) It was noticed that previous equations describing tension softening curve, obviously appropriate for the former researches and their experimental results, occurred not enough for experiments with complete elimination of secondary flexure of concrete

performed by automatically control gear system.

(2) The curves expressed by new formulas gave very good coefficient of correlation – around 0.999, very low chi-square parameter, nearly prediction of critical crack opening displacement and also appropriate shape.

(3) The best prediction of the data including all above conclusion seems to be Equation(7), like below:

$$\sigma_r(w) = 1 - a \cdot \exp(-b \cdot w) + \frac{1}{c \cdot w + d} + a - \frac{1}{d}$$

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