論文 THE SIMPLIFIED TRUSS MODEL FOR EVALUATING SHEAR CARRYING CAPACITY OF PRESTRESSED CONCRETE BEAMS

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ABSTRACT: In this study, the simple method based on the truss model analysis for evaluating the shear carrying capacity of prestressed concrete (PC) slender beams without transverse reinforcement failed in shear compression mode is proposed. The simplified truss model includes the consideration of D-region due to the applied load and supports. For simplicity, the summation of the resistance at the ultimate stage due to the main arch member and the resistance due to the effect of prestressing force is assessed as the shear carrying capacity of beams. Based on this approach, the shear carrying capacity of PC slender beams without transverse reinforcement can be easily predicted with the satisfactory accuracy.
KEYWORDS: PC slender beams without transverse reinforcement, shear carrying capacity, simplified truss model, D-region

1. INTRODUCTION

With the high resistance when subjected to the applied load and the prevention of occurrence of cracks in the serviceability stage, prestressed concrete (PC) beams, currently, become one of the most important structural members in civil engineering field. Due to the prestressing forces in the beams, after the diagonal crack took place, the generation of flat arch action in order to provide the redistribution of resistance is usually observed even in case of a PC slender beam where its shear span to effective depth ratio, *a/d*, is greater than 2.5. The PC slender beams are typically observed to fail in shear compression mode of failure due to the occurrence of crushing of concrete in the vicinity area near the loading point.

In evaluation of the shear carrying capacity of PC slender beams, several methods have been proposed. For instance, according to the Japan Society of Civil Engineers (JSCE) Standard Specification [1], the decompression moment method, in which the effect of axial force due to the prestressing is considered, is recommended. However, since the effect of types of stress distribution is not considered, the scattering of predicted results is usually observed. For other empirical equations such as ones suggested in American Concrete Institute (ACI) code [2], the shortcoming of this method is that the effect of prestressing is not clearly expressed and the comparatively conservative results are predicted.

In this study, the simplified truss model with a small number of degree of freedom including the effects of prestressing is proposed to evaluate the shear carrying capacity of PC slender beams without transverse reinforcement. As the superior points of this method, not only the simplicity in assessing, the effect of D-region where the flow of stress is comparatively complicated due to the applied load and supports [3] is also considered in order to provide the reasonable results.

2. SIMPLIFIED TRUSS MODEL ANALYSIS

Figure 1 shows the schematic diagram of the simplified truss model for analyzing the shear carrying capacity of PC slender beams without transverse reinforcement. The proposed model is divided into 3 regions, which are D-region due to the applied load, D-region due to the support, and B-region where the flow of stresses is uniform, by using the parameters m and n mentioned in **Fig. 1**. The explanations of determination of values of m and n will be provided in the next section. The modeling of concrete members and reinforcement are shown as in **Fig. 1**.

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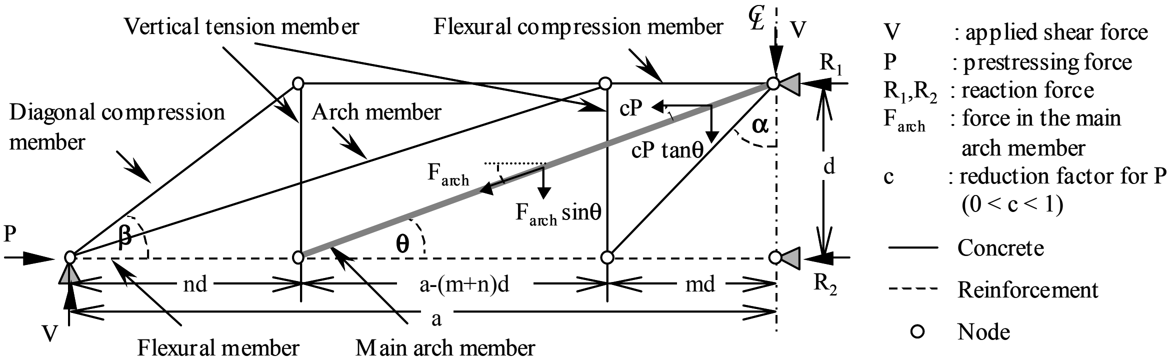

Figure 1 The simplified truss model for PC slender beam without transverse reinforcement

For the simplicity in calculation, this proposed model is modeled with a small number of degrees of freedom. From the experiments, even though the diagonal cracks occurred, a PC slender beam is still able to resist the applied load by the arch action until the ultimate stage. Therefore, the main arch member colored in the gray bold line in **Fig. 1** is modeled to represent the diagonal crack direction and the strut in the arch action. By considering the free body along the diagonal crack, the summation of the resistance at the ultimate stage due to the main arch member, F_{arch}, and the resistance due to the prestressing force, P, is considered as the shear carrying capacity of a PC slender beam. Here, the resistance due to the vertical tension concrete member along the diagonal crack and the dowel action are comparatively small and assumed to be neglected at the ultimate stage. The effect of prestressing force transferring to the upper portion of the beam at the ultimate stage is simply considered with the reduction factor, c. From **Fig. 1**, therefore, the shear carrying capacity of the PC slender beams without transverse reinforcement, V_{max}, can be expressed as **Eq. 1**.

$$V_{\max} = F_{arch} \sin\theta + cP\tan\theta \tag{1}$$

where θ is the inclination of the main arch member.

From **Eq. 1**, the shear carrying capacity can be obtained when F_{arch} is substituted in terms of $\eta f_c' \times A_{arch}$ where A_{arch} is the cross-sectional area of the main arch member and η is the compression softening parameter at the ultimate stage. Due to the existence of cracks in concrete at the ultimate stage, η is necessary to be taken into account. Consequently, V_{max} can be rewritten as

$$V_{\max} = \eta f_c' A_{arch} \sin\theta + cP\tan\theta \tag{2}$$

The values of m and n in the model, A_{arch} and η required in **Eq. 2** should be determined first.

3. DETERMINATION OF PARAMETERS

3.1 DETERMINATION OF m AND n

In this study, in order to find out the tendency of the flow of stresses in beams, the parametric study based on the elastic finite element analysis in DIANA system was performed. In the parametric study, the values of a/d, d, f_c', P and types of stress distribution, are considered as the parameters. The values of a/d were changed from 2.5 to 4.5 with the variation of d from 400 to 1400 mm. The value of the beam's width is set to be constant at 200 mm. The compressive strength of 30 or 70 MPa was applied as the representative of normal or high strength of concrete. The variation of values of P is in the range of 200- 500 kN. From the value of prestressing force and the location of applied axial load, the upper and lower fiber stresses (σ_u and σ_l, respectively) are calculated and correspondingly vary from -7~6 and 1~15 MPa. As shown in **Fig. 2**, the types of stress distribution can be categorized into 4 types, such as triangle with minus (tension) upper fiber stress, triangle with zero

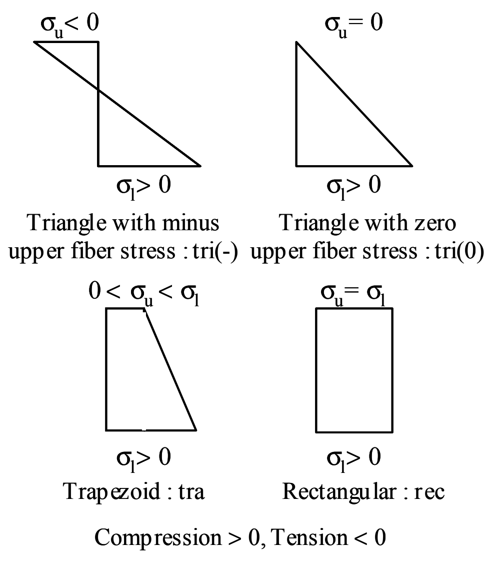

Figure 2 Types of stress distribution due to the prestressing force

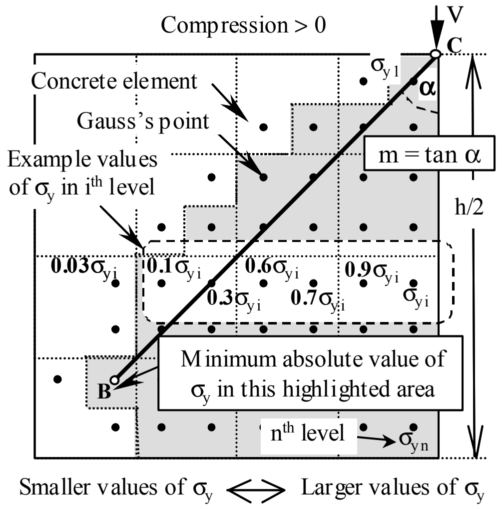

Figure 3 Determination of parameter m

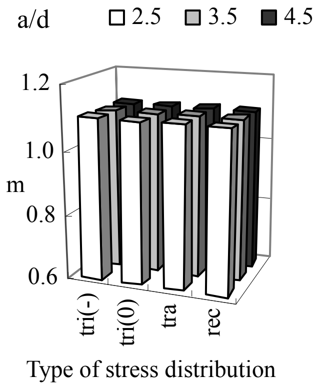

Figure 4 Effects of a/d on m (d =400 mm, P = 300 kN and f_c' = 30 MPa)

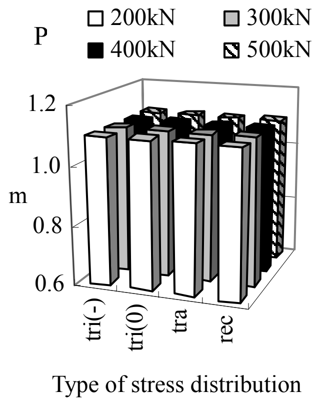

Figure 5 Effects of P on m (a/d = 3.5, d = 400 mm and f_c' = 30 MPa)

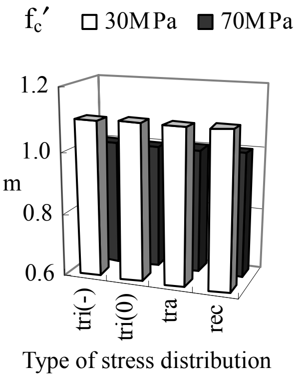

Figure 6 Effects of f_c' on m (a/d = 3.5, d = 400 mm and P= 300 kN)

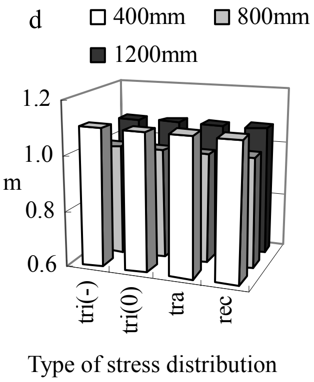

Figure 7 Effects of d on m (a/d = 3.5, P = 300 kN and f_c' = 30 MPa)

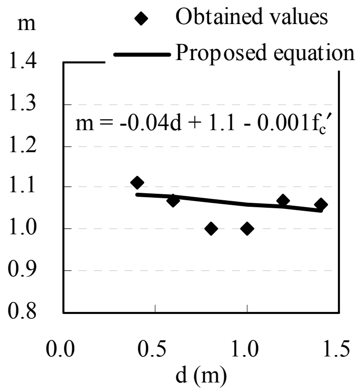

Figure 8 Proposed equation for evaluating m

upper fiber stress, trapezoid, and rectangular. In order to determine the representative flow of stresses in D-region as the direction of diagonal compression member, the clear stress flows are required and they can be obtained by adopting the shear force, which is equal to the applied prestressing force, to the elastic analysis. The compressive stresses in vertical direction, σ_y, were considered for determining the value of *m* as demonstrated in **Fig. 3**. In the range from the loading point to the middle height of the beam, the Gauss's points, in which the values of their σ_y are greater than 10% of the maximum value in each horizontal level, were considered and marked as the example values of σ_y in ith level, σ_{yi}. The area containing these Gauss's points was supposed to be D-region as highlighted in gray (**Fig. 3**). In this example, the point *B* is supposed to be Gauss's point with the minimum absolute value of σ_y in this highlighted area. The line connecting the loading point (*C*) and the Gauss's point *B* (Line *BC*) was considered. The direction of this line was assumed to be the direction of the diagonal compression member as shown in **Fig. 1**. The value of *m* can be calculated to be equal to tanα as expressed in **Fig. 3**. The values of *n* can also be assessed in the same manner as in case of *m* in the range from the support to the middle height of the beam. The value of *n* is equal to cotβ, where β is an angle declared in **Fig. 1**.

As depicted in **Figs. 4** and **5**, the values of *m* do not vary on the values of *a*/*d* and *P*. Whereas, the values of *m* are found to be depending on the values of f_c' and *d* as shown in **Figs. 6** and **7**. The value of *m* in case of high compressive strength decreases about 0.1 compared with the normal one as depicted in **Fig. 6**. Moreover, it should be noted that the types of stress distributions due to the same conditions of *a*/*d*, *P*, f_c' and *d* do not effect to the value of *m*. The equation for estimating the values of *m* based upon *d* and f_c' is simply proposed as expressed in **Eq. 3** (**Fig. 8**).

$$m = -0.04d + 1.1 - 0.001f_c' \quad \text{(units of } d \text{ is in meter and } f_c' \text{ is in MPa)} \quad (3)$$

The evaluation of values of *m* is also expressed here even though the value of *m* is an unnecessary factor

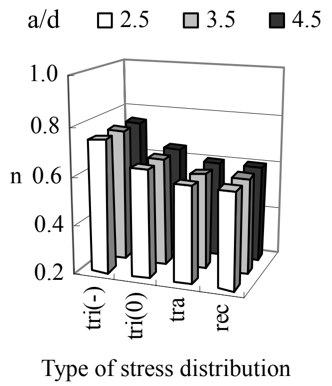

Figure 9 Effects of a/d on n (d = 400 mm, P = 300 kN and f_c' = 30 MPa)

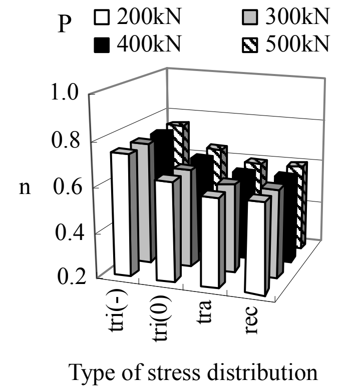

Figure 10 Effects of P on n (a/d = 3.5,d = 400 mm and f_c' = 30 MPa)

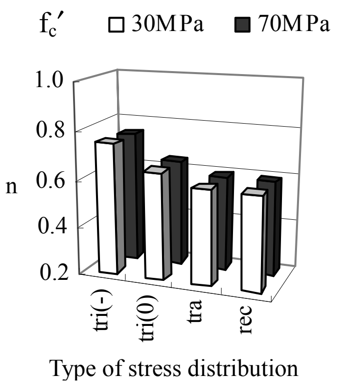

Figure 11 Effects of f_c' on n (a/d = 3.5, d = 400 mm and P= 300 kN)

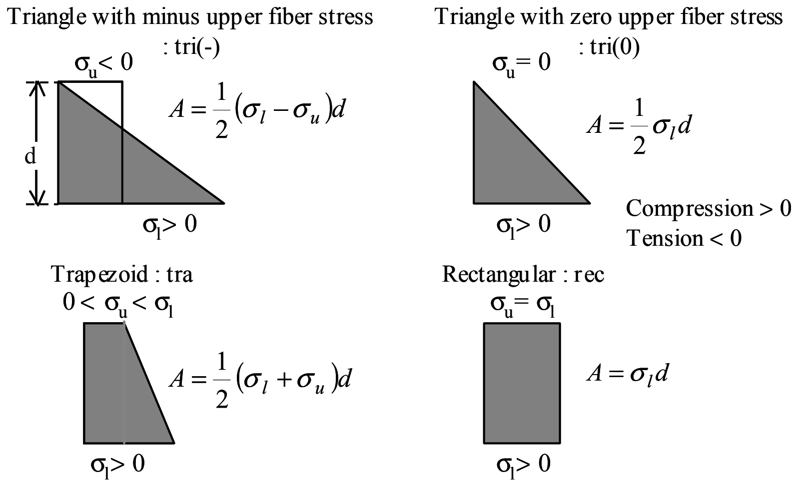

Figure 12 Definition of reference area; A

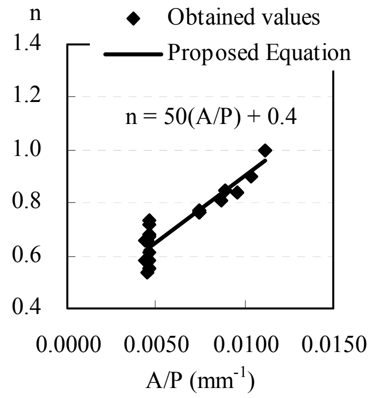

Figure 13 Proposed equation for evaluating n

in calculating the shear carrying capacity of the beams. However, the value of m is essential in cases of the assessing of resistance at the diagonal cracking.

It was found that there is no difference of values of n in cases that a/d, P and f_c' are varied for the same value of d as illustrated in **Figs. 9**, **10** and **11**, respectively. In the same condition of P, the stress distribution of triangle with minus upper fiber stress yields the largest compressive lower fiber stress and shear carrying capacity. From **Eq.2**, it is obvious that the greater value of n is, the greater shear carrying capacity will be predicted. Thus, this tendency is corresponding to the tendency of values of n as shown in **Figs. 9-11**. Here, the reference area, A (the shadowed area in **Fig. 12**), which is equal to the area under the stress distribution, was considered. It is found that the values of n can be linearly estimated in terms of the ratio of the reference area and the applied prestressing force as expressed in **Eq. 4** (**Fig. 13**).

$$n = 50(A/P) + 0.4 \qquad \text{(unit of } A/P \text{ is in mm}^{-1}) \tag{4}$$

3.2 DETERMINATION OF COMPRESSION SOFTENING PARAMETER (η)

In order to assess the reasonable values of η, the equation based on the consideration of the transverse tensile strain proposed by Vecchio and Collins [4] is generally used. In this proposed model, more complicated process is required to derive the value of transverse tensile strain. For the other expressions based on the value of f_c', the underestimation of values of η is usually observed, especially, in case of high strength concrete. Therefore, the value of compression softening parameter at the ultimate stage η is assumed to be 0.5 for all cases as the uncomplicated and practical standard.

3.3 DETERMINATION OF CROSS-SECTIONAL AREA OF MAIN ARCH MEMBER (A_{arch})

From the assessing of shear carrying capacity of RC deep beams ($a/d \leq 1$), which also fail in shear compression mode, proposed by Niwa [5], the thickness of the arch member is approximated to be

$(r$+0.3$d)$sinθ, where r is the width of bearing plate and θ is the inclination of the arch member. Even though the arch member in PC slender beams performs as the bulging strut at the ultimate stage, due to the geometric condition, the thinner arch compared with one of RC deep beams is created. For simplicity, the small extra thickness is used as 0.1d and then the thickness of the main arch member is considered to be equal to $(r$+0.1$d)$sinθ. For the case that the rollers were directly applied for the loading, r is assumed to be 0. The values of A_{arch} can be evaluated from **Eq. 5**, where b is the width of a PC slender beam.

$$A_{arch} \quad = \quad b(r + 0.1d)\sin\theta \qquad (5)$$

4. CALCULATED RESULTS AND DISCUSSION

To verify the applicability of this proposed approach, the experimental results of normal and high strength PC slender beams without transverse reinforcement are applied for comparing with the calculated results in this study. The details of the specimens and the comparisons between the experimental and calculated results are tabulated in **Table 1** [6-9]. All specimens failed by shear compression mode of failure excluding specimens 4-6,4-7,1-9,and 1-10 tested by Sato, et al. [7], in which the failure mode was not evidently stated. Since these specimens were subjected to high prestressing forces, the failure mode might be the shear compression. In the tests of Arthur [8], since the bearing plates were used but their widths are not mentioned in the reference, the values of r are substituted with 50 mm as the possible values. The calculated results can be computed by using the proposed values of n, η and A_{arch} to **Eq. 2**. As a simple application for the primary study, c is assumed to be 1 in the calculation. The stress distributions of all cases are categorized in the triangle with minus (tension) upper fiber stress.

It is apparent that the correlation of the calculated results with the test data is in the acceptable range,

Table 1 Comparisons between the experimental and calculated results of PC slender beams

Researcher (Shape of cross section)	Specimen	a/d	b (mm)	f_c' (MPa)	σ_u* (MPa)	σ_l* (MPa)	m	n	r (mm)	Shear carrying capacity (kN)		$V_{exp.}/V_{cal.}$	Arch angle (°)
										$V_{exp.}$	$V_{cal.}$ (η=0.5)		
PWRI (Rec) [6]	H3-35-30	3.0	200	92.0	-1	3	1.0	0.8	100	246	252	0.97	24
	H3-35-60	3.0	200	86.3	-2	6	1.0	0.8	100	284	278	1.02	24
	H3-35-90	3.0	200	70.3	-3	9	1.0	0.8	100	295	281	1.05	24
	H3-55-30	3.0	200	84.0	-1	3	1.0	0.9	100	228	295	0.77	26
	H3-55-60	3.0	200	78.3	-2	6	1.0	0.9	100	361	333	1.08	26
	H3-75-30	3.0	200	88.5	-1	3	1.0	0.9	100	345	354	0.97	26
	H3-75-60	3.0	200	87.4	-3	6	1.0	1.0	100	436	449	0.97	27
	H3-95-60	3.0	200	71.4	-2	6	1.0	0.9	100	586	455	1.29	26
	L3-35-30	3.0	200	49.7	-1	3	1.0	0.8	100	194	154	1.26	24
	L3-35-60	3.0	200	40.6	-2	6	1.0	0.8	100	202	172	1.17	24
Sato (Rec) [7]	4-6	3.3	150	40.1	-5	14	1.0	0.9	50	170	162	1.05	23
	4-7	3.3	150	43.6	-5	14	1.0	0.9	50	167	165	1.01	23
	4-10	3.3	150	40.9	-2	5	1.0	1.0	50	99	85	1.16	24
	4-12	3.3	150	39.7	-5	14	1.0	0.9	50	162	161	1.01	23
Sato(T) [7]	1-9	3.3	150	43.7	-5	23	1.0	0.9	150	239	297	0.80	23
	1-10	3.6	150	40.2	-5	23	1.0	0.9	150	270	244	1.11	20
Arthur (I) [8]	A17	4.6	51	38.6	-3	13	1.1	1.1	50	32	37	0.86	16
	A18	4.6	51	41.4	-3	13	1.1	1.2	50	33	38	0.87	16
	A19	4.6	51	38.6	-3	12	1.1	1.1	50	31	35	0.89	16
	A22	4.6	51	39.7	-3	12	1.1	1.1	50	32	36	0.89	16
	D2	3.9	51	45.2	-4	14	1.0	1.2	50	50	67	0.75	20
Hamada (Rec)[9]	45LC-0	3.5	200	47.0	-1	3	1.0	0.9	150	135	135	1.00	21
	60LC-0	3.5	200	58.8	-1	3	1.0	0.9	150	155	162	0.96	21
											AVE.	**1.00**	
											C.V.	**0.14**	

*: Compressive and tensile stresses are expressed in the positive and negative values, respectively.

with the ratio of experimental and calculated results having an average value (AVE.) of 1.00 and a coefficient of variation (C.V.) of 0.14. Nevertheless, somewhat difference of results, which is more than ±20% compared with experimental results, is found. It is noteworthy that even though the values of stress distribution are changed, the proposed method yields the satisfactorily predicted results. Moreover, not only in cases of PC slender beams with rectangular cross section, but the satisfactory results of shear carrying capacity of PC slender beams with I or T shaped cross section are also obtained based on this approach. However, in the further study, more experimental data as the targets in the calculation are required in order to confirm the applicability of this proposed approach.

It might be an overestimation that the total prestressing force is assumed to concentrate in the upper portion of the beam at the ultimate stage. This contribution is also considered to cover the extra resistances due to the concrete tension, the compression due to the flexural mechanism and the dowel action, which are neglected in this calculation. Nevertheless, more accurate evaluation of effects of prestressing force in terms of c is required to obtain more reliable prediction.

About the parameters m and n, although the values of these parameters are varied in the narrow range, the reasonable evaluation of these parameters is necessary for the accurate prediction. Therefore, the consideration of D-region in the model is realized to be the important process.

Additionally, the inclination of diagonal crack can be approximated as the values of angle of the main arch member and it can be computed from $\tan^{-1}(1/[(a/d)-n])$. It is noticed that inclinations of the main arch are slightly flat corresponding to their shape and considered to be in the possible range.

5. CONCLUSIONS

For PC slender beams without transverse reinforcement failed in shear compression mode, the equation for evaluating their shear carrying capacity has been proposed by assuming that the externally applied load is resisted by the contributions of the main arch member and the prestressing force in the simplified truss model. In this proposed method, the effects of D-region due to the applied load and supports in terms of parameters m and n are taken into the consideration. In addition, the compressive strength of concrete with compression softening parameter at the ultimate stage and the possible cross sectional area of the main arch member in terms of the width of bearing plate and effective depth are also considered as the significant parameters in the calculation. By applying this approach, the shear carrying capacity of normal and high strength PC slender beams without transverse reinforcement can be uncomplicatedly assessed with the adequate level of prediction compared with the experimental results.

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