論文 Simulation of Externally Prestressed Concrete Beams using the Deformation Compatibility of Cable

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ABSTRACT: In this study, the deformation compatibility of cable in the externally prestressed concrete beams is presented. Application of the developed equations in the numerical analysis of six examples (three beams with normally external cables, three beams with large eccentricities) is carried out to verify its accuracy. Effect of the cable eccentricity on the behavior of externally prestressed concrete beams is also investigated. The analytical results are compared with experimental data, good agreement is found.
KEYWORDS: external cable, deviator, unbonded, cable strain.

1. INTRODUCTION

External prestressing is defined as prestressing by the high strength cable, which is placed outside of the cross section and attached to the beam at some deviator points along the beam. In an external prestressing system, depending on the location of deviators, there are two kinds of the beam, namely, conventionally prestressed concrete (PC) beam with external cables (below referred as typical beam) and PC beam with large eccentricities (beam with large eccentricities). In the former, the deviators are located within the depth of cross section. In the latter, the deviators are located outside of the depth of cross section, above the top surface or under the bottom surface.

In the analysis of PC beams with external cables, questions always raises in the calculation of cable strain, because, there is no bonding between the concrete and the cable, thus the cable strain cannot be calculated at the critical section as in the conventional PC beams with bonded cables. For the analysis of the typical beams, an analytical methodology has been developed, based on the deformation compatibility between the concrete and the cable, and good agreement with the experimental data has been reported [3, 4].

In principle, the analysis of the beam with large eccentricities can use the analytical methodology for the typical beams. However, the only additional point should be considered in case of the beam with large eccentricities, namely, eccentricity of the cable due to the location of the external cable. The difference in the analysis of both kinds of the beam with external cables is that the overall deformation of the beam in terms of the concrete strain, which is usually used in the calculation of cable strain for the typical beam, cannot be used in the analysis of the beam with large eccentricities. Because, in case of the beam with large eccentricities, the cable portions are almost located outside of the depth of cross section, thus the concrete does not exist at the cable level.

Therefore, there is a need to have a computation method for the cable strain that should take into account the cable eccentricity, friction at the deviator and continuity of the structure, and the method can be used in the analysis of both kinds of the beams as above mentioned. To satisfy these conditions, the elongation of the cable must be in consistent with its deflection and to that effect, geometrical deformation of the cable must be correctly evaluated regardless of the deformed configuration of the beam. One of the solutions is to assume that the cable strain depends only on the deformation of the points, to which the cable is attached. It turns out that the cable strain depends on the total length variation of cable between the extreme ends. Therefore, formulation of the cable strain which is to be used in the analysis of the beams with external cables, will be presented below.

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2. REVIEW OF PREVIOUS STUDIES

In many studies [1~4, 9, 10], when structural behavior of the typical beam was investigated, most the analytical approaches are usually based on the assumptions in which the total elongation of the cable element must be equal to the total elongation of the concrete element at the cable level between the extreme ends. This can be expressed in the following equation:

$$\sum_{i=1}^{n} l_i \Delta\varepsilon_{si} = \int_{o}^{l} \Delta\varepsilon_{cs} dx \tag{1}$$

where $\Delta\varepsilon_{si}, \Delta\varepsilon_{cs}$ are the increments of cable strain and concrete strain at the cable level, respectively; l_i is the length of cable element under consideration; l is the total length of cable between the extreme ends. This assumption is considered to be the effective tools for the evaluation of cable strain in the analysis of the unbonded PC beams as well as typical beams, and good agreement with the experimental data has been reported [1~4, 9, 10].

Some researchers [1, 2, 7] extended this approach for the analysis of the beams with large eccentricities by the additional assumption of an imaginary concrete strain at the portions of cable, at which the concrete does not exist (see **Fig.1**). However, this extension seems to be limited because of difficulties in defining value of the imaginary concrete strain at the cable level. In the case, the prestressing cable has very large eccentricity compared to the height of cross-section, and the application of this assumption is not so easy to apply, evidently.

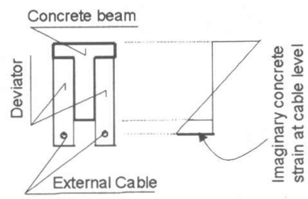


Fig.1 Imaginary concrete strain at the cable level

Virlogeux, M. [8] proposed another approach based on a geometrical compatibility of the external cable. Due to the rectilinear shape of external cable between the points, at which the cable attaches to the beam, the strain variation of cable can be defined on the basis of deformations of the attachment points. Therefore, the cable strain can be evaluated regardless of the deformed shape of the beam and it depends only on deformations of the deviator points. By using this concept, Eakarat, W. et al. [7] developed a computing program for the analysis of the simply supported beam with large eccentricities, and good agreement with experimental data has been reported. However, the effect of cable friction at the deviator did not consider in the author's calculation.

3. PROPOSED DEFORMATION COMPATIBILITY OF CABLE

Unlike the previous assumptions for the analysis of typical beam, for the beam with large eccentricities, instead of using the overall deformation of the beam, the only deformations at the extreme top of deviators are considered in formulation of the cable strain. In case of the cable being perfectly fixed at the deviators, the cable strain of each segment is independent from that of the others On the other hand, the cable can be allowed to slip at the deviators, the total elongation of cable must be equal to the total cable length variation of each segment and expressed as:

$$\sum_{i=1}^{n} l_i \Delta\varepsilon_{si} = \sum_{i=1}^{n} \Delta l_i \tag{2}$$

where $l_i, \Delta\varepsilon_{si}$ are the length and strain increment of cable element under consideration, respectively; Δl_i is a cable length variation of each element.

The cable length variation Δl_i can directly be derived from the deformation of deviators, for example the deviators 1 and 2 are shown in **Fig.2**.

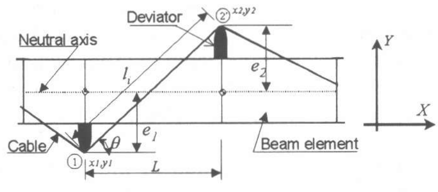


Fig.2 Arrangement of external cables

Before deformation, the coordinates of deviators 1 and 2 at the extreme top are x_1,y_1 and x_2,y_2 , respectively. After deformation, the deviators 1 and 2 shift to new positions and their coordinates are $x_1 + \Delta x_1, y_1 + \Delta y_1$ and $x_2 + \Delta x_2, y_2 + \Delta y_2$. After some manipulations with neglecting the high order terms, the cable length variation Δl_i can be calculated as:

$$\Delta l_i = \cos\theta(\Delta x_2 - \Delta x_1) + \sin\theta(\Delta y_2 - \Delta y_1) \tag{3}$$

where θ is the angle of the external cable to the horizontal line; $\Delta x_1, \Delta x_2$ and $\Delta y_1, \Delta y_2$ are called as incremental displacements in the horizontal and the vertical directions at the deviators 1 and 2, respectively.

Based on the displacement functions for the beam element [3,4], these increments of displacement can be defined. Therefore, the cable length variation between the deviators 1 and 2 can be rewritten in a contractile form as:

$$\Delta l_i = [A]\{d^*\} \tag{4}$$

where the matrix [A] can be found in reference [3, 5], $\{d^*\}$ is the nodal displacement vector at the extreme top of deviators. This nodal displacement vector can be expressed in terms of the nodal displacement vector for the beam element as:

$$\begin{Bmatrix} u_1^* \\ v_1^* \\ \theta_1^* \\ u_2^* \\ v_2^* \\ \theta_2^* \end{Bmatrix} = \begin{bmatrix} 1 & 0 & e_1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & e_2 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \end{Bmatrix} \qquad \{d^*\} = [B]\{d\} \tag{5}$$

where e_1, e_2 are the eccentricities of cable at the deviators 1 and 2, respectively.

Substituting Eq.(5) into Eq.(4) to obtain the cable length variation between the deviators 1 and 2 which directly relates the nodal displacement vector for the beam element and expressed as:

$$\Delta l_i = [A]\{d^*\} = [A][B]\{d\} \tag{6}$$

Substituting Eq.(6) into Eq.(2), the total elongation of cable can be expressed as:

$$\sum_{i=1}^{n} l_i \Delta\varepsilon_{si} = \sum_{i=1}^{n} [A][B]\{d\} \tag{7}$$

From Eq.(7), once again it can be seen that the elongation of cable is a function of deformation of the beam. Therefore, increasing the beam deformation under the applied load is in relative change of the cable elongation. If the deformed shape of the beam (displacement of every point of the beam) is known, and then the deformation of the cable can be determined too.

4. NUMERICAL EXAMPLES

A total of six PC beams with external cables, which were tested by Saitama University [1] and Sumitomo Co.[9, 10], is analyzed in this section. Four beams (A1, A2, B1, B2) are the simply

Table 1 The tested beam variables and their materials

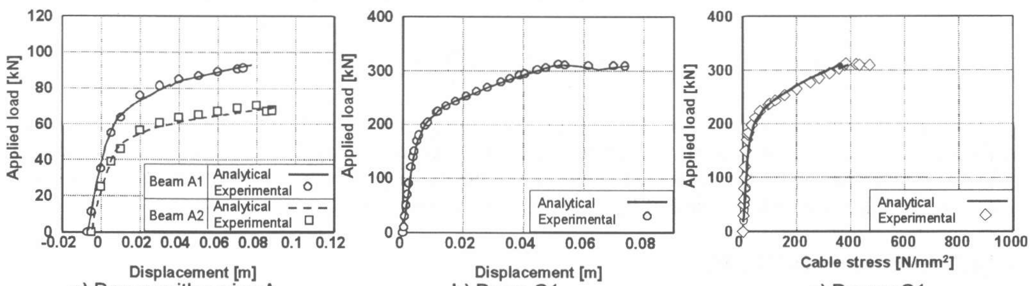
No	Descriptions of specimens	The depth of cable mm	Prestress force, kN	Concrete strength, N/mm^2	Cable strength, N/mm^2	
					F_{psy}	F_{psu}
A1	Simply supported, typical beam	250	265.0	35.0	1500	1750
A2	Simply supported, typical beam	250	177.0	35.0	1500	1750
B1	Simply supported beam with large eccentricities.	375	118.0	35.0	1500	1750
B2	Simply supported beam with large eccentricities.	375	177.0	35.0	1500	1750
C1	Typical beam with two continuous span	445/75*	382.0	42.4	1600	1900
C2	Two span continuous beams with large eccentricities.	750/-320*	164.0	42.4	1500	1750

* at the midspan section / the center supported section, (-) deviator is located on the top surface of cross section.

supported beams with a flanged cross-section, whereas the other two beams (C1, C2) are two span continuous beams with a rectangular cross-section. The beams with series A are the typical beams with the depth of cable from the top surface of 250.0 mm, and the beams with series B are the beams with large eccentricities with the depth of cable of 375.0 mm. The beams A1, B1 and the beams with series C were designed to achieve the same ultimate strength, but with different prestressing force. While, the beams A2 and B2 were prestressed by the same amount of the prestressing force at the initial prestressing stage in order to compare the increase in the load capacity of the beam. At the initial prestressing stage, the cables were stressed approximately 50% of the ultimate strength of cable It should be noted that the beam B2 was additionally prestressed by one internally bonded cable at the top flange to prevent the expected cracks due to prestressing. Material properties are shown in **Table 1** Two loading points of the applied load are provided on each span with the symmetrical loading condition. A more detailed test setup and geometrical dimensions of the tested beams can be found in references [1, 9, 10].

4.1 GENERAL DISCUSSION OF THE ANALYTICAL RESULTS

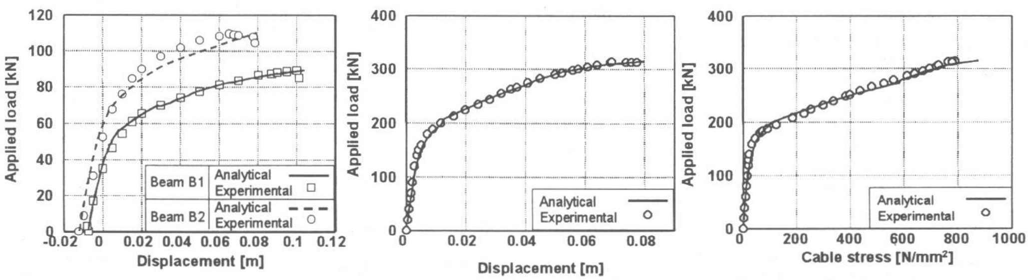
Fig.3a, b and **Fig.4a, b** represent the predicted load-displacement relationships of the beams in comparison with the experimental data. From these figures, it can be seen that the displacement responses are predicted very well, and are in full agreement with the experimental data for the simply supported beams as well as for the continuous beams. At the ultimate state, the maximum values of the applied loads are 92.9 kN, 69.1 kN, 89.1 kN, 110.2 kN, 310.8 kN and 310.2 kN, corresponding to the maximum displacements of 73.0 mm, 83.6 mm, 102.0 mm, 78.0 mm, 50.0 mm and 69.8 mm for the beams A1, A2, B1, B2, C1 and C2, respectively. Although, dispite of some discrepancy in the load-displacement realtionship of the beam B2, the analysis captures the trend of the increase in dispalcement against the applied load untill the ultimate state. Nevertheless, the predicted load capacity is not so far from the experimenal observation, the precision of the analytical results is still quite good, and the anlytical method can acurately show the general behavior of PC beams with external cables.



a) Beams with series A b) Beam C1 c) Beams C1

Fig.3 Behavioral characteristics of the typical beams

Fig.3c and **Fig.4c** show the increase of cable stress against the applied loads for the beams C1 and C2, respectively. It can be seen that for the beam C1, the stress in the external cable increases only slowly so that when the crushing strain has been reached in the concrete, the stress in the cable is



a) Beams with series B b) Beam-C2 c) Beam-C2

Fig.4 Behavioral characteristics of the beams with large eccentricities far below its ultimate strength. The prestressing cable undergoes small stress and remains in the elastic range. While, for the beam C2, the stress in the cable increases very fast and almost proportionally increases with further increasing the applied load after the decompression. Therefore, for the beams with large eccentricities, the external cable has usually yielded before reaching the maximum value of the applied load. From these figures, it is apparently shown that the predicted responses have a good accuracy in comparison with the experimental data.

The analytical results of the important parameters in comparison with the experimental data are summarized in **Table 2**. From this table, it is shown that the higher load capacity and the lower deflection can be obtained in the beam A1 (92.9 kN and 73.0 mm) with the higher prestressing force at the prestressing stage (265.0 kN) than that in the beam A2 (69.1 kN and 83.6 mm with the prestressing force of 177.0 kN). The same behavior can be found for the beams with large eccentricities B1 and B2 (see **Fig.4a**).

Table 2 Summary of experimental data and analytical results

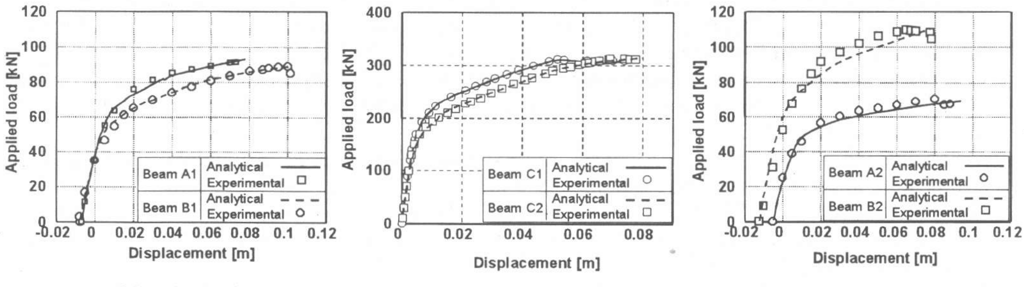
No	Ultimate load kN			Stress increase in cable, N/mm^2			Ultimate deflection mm		
	Exp.	Calc.	Calc/Exp	Exp.	Calc.	Calc/Exp	Exp.	Calc.	Calc/Exp
A1	91.2	92.9	1.018	357.0	348.0	0.975	72.8	73.0	1.003
A2	70.3	69.1	0.983	417.0	405.0	0.971	88.1	83.6	0.949
B1	88.6	89.1	1.005	698.0	702.0	1.006	101.3	102.0	1.007
B2	109.6	110.2	1.005	552.0	545.0	0.987	78.3	78.0	0.996
C1	308.0	310.8	1.009	370.0	366.0	0.989	48.0	50.0	1.042
C2	315.0	310.2	0.985	780.0	810.0	1.038	69.0	69.8	1.012

4.2 EFFECT OF CABLE ECCENTRICITY ON THE BEAM BEHAVIORS

To show the effect of cable eccentricity, the predicted results of the typical beams and the beams with large eccentricities will be compared to each other and discussed in this section.

Fig.5b shows the load-displacement responses of the beams C1, C2 against the applied load. It can be seen that in the elastic zone, both kinds of the beams with external cables behave the same and indicating no influence of the arrangement of the external cable. However, in the non-elastic cracked zone, the displacement response deviates from each other. For a given the applied load, the larger deflection is found in the beam with large eccentricities than that in the typical beam. It is clearly shown that in the non-elastic cracked zone, the eccentricity of cable has a significant effect on the displacement response of the beam. Even though, the ultimate load capacity of beams C1 and C2 is nearly the same, but the maximum displacements are significantly different at the ultimate state, and they are about 50.0 mm and 69.8 mm for the analytical models, respectively. The difference in displacement could be mainly attributed to the lower prestressing force applied in the beam with large eccentricities at the prestressing stage than that in the typical beam. Moreover, the external cable in the beam with large eccentricities reaches the yielding strength before the ultimate load capacity of the beam. The same behavior for the simply supported beams A1 and B1 can be seen in **Fig.5a**.

Fig.5c shows a comparison between the analytical results and the experimental data for the



a) Beams A1-B1 b) Beams C1-C2 c) Beams A2-B2

Fig.5 Effect of cable eccentricity on the behavior of beams with external cables beams A2 and B2, which were designed by the same amount of the prestressing force at the prestressing stage. It clearly shows that the beam B2 posesses the bigger camber than the beam A2 before the application of the external load , which means that the beam B2 is comparatively prestressed more. As a result, the load capacity increases significantly, and it increases about 60% of the load capacity of the beam A2 at the ultimate load state. Moreover, the maximal displacement of beam B2 is slightly smaller than the displacement of the beam A2 (78.0 mm compared to 83.6 mm).

Fig.5d shows the increase of cable stress against the applied load for the beams C1 and C2. It can be clearly seen that for a given the applied load, the stress increase in the cable of the beam C2 is greater than that in the beam C1. This is because the stress increase in the cable is a function of the total elongation of cable, and essentially depends on the deformation of the beam. Hence, the large deflection in the beam C2 would contribute to great stress in a cable than that obtained in the beam C1. Moreover, at the prestressing stage, the beam C2 was prestressed with lesser prestressing force, which was approximately 45% of the prestressing force in the beam C1. For the beam C2, the cables have yielded when the applied load reaches about 285.0 kN, whereas they did not yield even at the ultimate state for the beam C1. At the ultimate state, the stress increases in the cable are about 366.0 N/mm^2 and 807.3 N/mm^2 for the beam C1 and the beam C2, respectively.

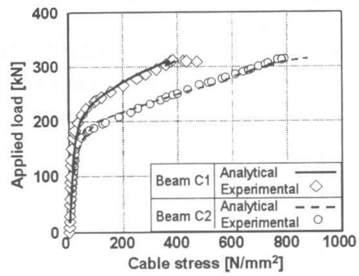


Fig.5d Beams A2-B2

5. CONCLUSIONS

The formulated equation for cable strain is in the general form, and it can satisfactorily predict the structural behavior of the typical beam as well as the beam with large eccentricities. The predicted responses are in good agreement with the experimental data. A big eccentricity of cable induces a great strain variation in the cable. Therefore, in the same conditions, the increase of cable stress in the beam with large eccentricities is usually greater than that in the typical beam at the certain loading stage after the decompression.

REFERENCES

1. Aravinthan, T., Mutsuyoshi, H., Niitsu, T., Chen, A., "Flexural Behavior of Externally Prestressed Concrete Beams with Large Eccentricities", Proceeding of JCI, 1998, Vol.20, No3, pp.673-678.
2. Aravinthan, T., Mutsuyoshi, H., Hamada, Y., Watanabe, M., "Experimental Investigation on The Flexural Behavior of Two Span Continuous Beams with Large Eccentricities", Proceeding of JCI, 1999, Vol.21, No3, pp.961-966.
3. Diep, B.K., "Non-Linear Analysis of Externally Prestressed Concrete Beams Considering Shear Deformation", Master thesis, Nagoya University, 2000.
4. Diep, B.K., Tanabe, T. "Analysis of Externally Prestressed Concrete Continuous Beams Considering Shear Deformation", Transaction of JCI, Vol.21, 1999, pp.307-314.
5. Diep, B.K., Tanabe, T., Umehara, H., "Study on Behavior of Externally Prestressed Concrete Beams Using the Deformation Compatibility of Cable" Accepted for Publication of JSCE Journal.
6. Eakarat, W., Mutsutoshi, H., Aravinthan, T., Watanabe, M., "Effect of Loading Arrangement on Flexural Behavior of Externally PC Beams with Large Eccentricities", Proceeding of The First International Summer Symposium in Tokyo, JSCE, 1999, pp.287-290.
7. Eakarat, W., Mutsuyoshi, H., Aravinthan, T., Watanabe, M., "Analysis of The Flexural Behavior of Externally Prestressed Concrete Beams with Large Eccentricities", Proceeding of JCI, 2000, Vol.22, No3, pp.817-822.
8. Virlogeux, M., "Non Linear Analysis of Externally Prestressed Structures", International Symposium in Jerusalem-Israel, 1988, pp.309-340.
9. Umezu, K., Fujita, M., Tamaki, K., Yamazaki, J., "Ultimate Strength of Two Span Continuous Beams with External Cable", Proceeding of 5th Symposium on Development in Prestressed Concrete, 1995, pp.303-308.
10. Umezu, K., Fujita, M., Tamaki, K., Yamazaki, J., "Study on Ultimate Strength of Two Span Continuous Beams with External Cable", Proceeding of JCI, 1995, Vol.17, No2, pp.743-748.