# 論文 Nonlinear FEM Model and Design Formula for Externally Bonded Steel Plates for Shear Enhancement of RC Beams

# Bimal Babu ADHIKARY\*1 and Hiroshi MUTSUYOSHI\*2

ABSTRACT: A two dimensional nonlinear FEM analysis is done to simulate the behavior of RC beams with web-bonded continuous steel plates employing a special interface element. Simulations are done for tested beams and validated the model. A parametric study is conducted with parameters such as concrete strength, plate thickness, plate depth to beam depth ratio and internal shear reinforcement ratio. A formula to compute the shear strength of such plate-bonded beams is proposed and the validity is checked against the results from parametric study and experiments. The proposed formula predicts the ultimate shear strengths of such plate bonded RC beams very well.

KEYWORDS: Epoxy bonding; Nonlinear FEM model; RC beams; Shear enhancement; Shear strength; Shear strengthening; Steel plates

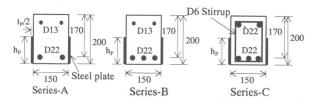
## 1. INTRODUCTION

Deterioration of existing concrete structures is a worldwide problem today. Besides, in many countries, design codes have been upgraded now and then due to increased live load requirements. Thus, strengthening and retrofitting of existing concrete structures has become a mainstream construction activity in recent times. Bonding of steel plates or Fiber Reinforced Plastics (FRP) composites with suitable epoxy adhesive on external surface of structures is the most versatile and widely used technique for strengthening RC beams. Generally, the soffit (bottom) bonding is done for flexural strengthening, while web bonding is preferred for shear strengthening. Bonding of continuous steel plates on beam webs is one of the highly effective methods, since it can provide both the shear and flexural enhancement for concrete beams. Authors have studied the effectiveness of this type of strengthening method for shear and found that about 35-84% increment could be obtained in ultimate shear strength of plate bonded beams [1, 2]. However, due to the lack of extensive experimental database and analytical results, the effects of various factors such as plate depth across the section, plate thickness and concrete strength on shear strength of plate bonded beams are not clear yet. So far, there exists no design formula for computing the shear strength of RC beams with web-bonded continuous steel plates. Besides, there is a dearth of an appropriate numerical tool that can simulate the overall behavior of such strengthened RC beams up to final failure. This paper presents an outline of a nonlinear FEM model to analyze the RC beams with webbonded steel plates and simulation results of some beams tested by the authors. Further, it presents the results of a parametric study conducted on RC beams with various parameters such as plate depth/beam depth ratio, plate thickness, concrete strength and internal shear reinforcement ratio. Finally, a design formula to compute the shear strength of beams with web-bonded continuous steel plate is presented. Comparisons between the shear strengths computed using the proposed formula and FEM as well as the experimental results are made.

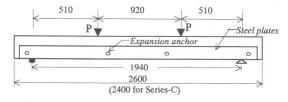
<sup>\*1</sup> Department of Civil and Env. Engineering, Saitama University, Research Assoc., Member of JCI

<sup>\*2</sup> Department of Civil and Env. Engineering, Saitama University, Prof., Member of JCI

## 2. OVERVIEW OF EXPERIMENTS



a. Beam cross-sections (dimensions in mm)



b. Beam loading details (dimensions in mm)

Fig. 1 Beam details

The experimental program consisted of testing two sets of beams, differing in the provision of internal shear reinforcement. The details of beams are shown in Fig. 1. In the first set without shear reinforcement, a total of twelve beams in two series A and B were tested [1]. Beams A-1 and B-1 were control beams without steel plates, while the other beams had steel plates bonded to them. In the second set (Series-C), total five beams were tested [2]. The beam size used was same except the length was 2400 mm. The beams were provided with 2-legged D6 (deformed bar of 6 mm diameter) hooked stirrups at 170 mm center to center in both shear spans. Beam C-1 was control beam without steel plates while the other beams were bonded with steel plates. All beams were tested under four-point loading over a span of 1940 mm. The shear span to effective depth ratio was a constant 3 in all tests.

#### 3. TWO DIMENSIONAL FEM ANALYSIS

A sixteen-node adhesive interface element is employed to model the concrete-adhesive-steel joint. Stresses and relative displacements are evaluated between the two contact surfaces. The element is formulated on the basis of relative nodal displacements of steel and concrete elements surrounding the adhesive interface element, behavior of which is governed by shear stiffness in two tangential directions. To simulate the effect of anchor bolts that were used in the tests, bolt crossing joint (BOCJ) element is formulated, which is modeled as two orthogonal springs. For reinforced concrete and external steel plates, eight-node conventional finite elements are chosen. The RC plate element model [3] is used for reinforced concrete and failure criteria adopted are as suggested by Shawky and Maekawa [4]. Steel plates are modeled as isotropic elastic-perfectly-plastic material with von Mises's yield criterion. The shear behavior of concrete-epoxy-steel interface is assumed to be linear elastic till failure. Shear stiffness of interface is taken as 60 MPa/mm [5] in both tangential directions. The cracking and local debonding of interface is modeled by the maximum effective strength criteria. The value of cut-off strength is taken as 5 MPa in this analysis.

Due to symmetry of geometry and loading pattern, only half portion of each beam is analyzed. At each coordinate, three layers of coincident nodes are defined. The middle layer is used for the reinforced concrete, whereas outside nodes are used for the steel plates. The steel plates are connected with the RC elements with adhesive interface elements. The anchor bolts are simulated with the use of BOCJ elements that connect two opposite nodes of interface at the location of the bolt.

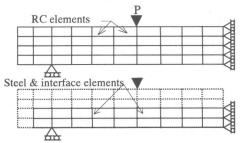
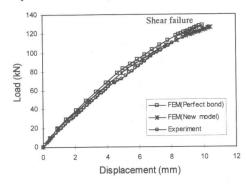


Fig. 2 Finite element mesh

Another set of analysis is also conducted with perfect bond assumption for the sake of comparison, where no interface elements are used. Finite element mesh used for beams is shown in Fig. 2.

# 4. FEM RESULTS AND COMPARISON WITH EXPERIMENTS

The ultimate loads and failure modes as predicted by FEM analysis along with the experimental results for some selected beams are presented in **Table 1**. It is seen that the FEM analysis with interface element predicts the ultimate failure loads and failure modes of all beams quite accurately. However, analysis with perfect bond assumption little overestimates the failure loads. For particular beam C-5, the failure mode predicted from perfect bond analysis is flexural one, which is erroneous. It is thus confirmed that though the perfect bond analysis is satisfactory in general, but the analysis with interface element is superior in overall performance. **Fig. 3** and **Fig. 4** show the load versus mid-span deflection relationships for some typical beams from experiments and FEM analysis. These figures clearly show the accuracy of model with interface element over that of the perfect bond analysis.



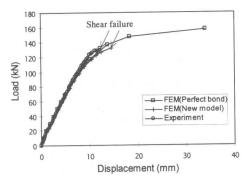


Fig. 3 Load displacement relationship beam C-4 Fig. 4 Load displacement relationship beam C-5

Table 1 Failure loads from experiment and FEM analysis

Beam	Exp.(kN)/	FEM (1)(kN) /	FEM (2)(kN) /	FEM (1) /	FEM (2) /
no.	failure mode	failure mode	failure mode	Exp.	Exp.
A-2	55.2 / shear	59.8 / shear	56.8 / shear	1.08	1.03
A-3	62.2 / shear	64.7 / shear	62.7 / shear	1.04	1.01
A-5	83.4 / shear	84.3 / shear	82.4 / shear	1.01	0.99
A-7	93.5 / flexure	107.6 / flexure	104.6 / flexure	1.15	1.12
B-2	58.8 / shear	65.7 / shear	62.7 / shear	1.12 Mea	1.00
B-3	74.4 / shear	76.5 / shear	73.5 / shear	1.03 = 1.0	07  0.99  =1.02
B-4	74.4 / shear	79.4 / shear	77.4 / shear	1.07 = 0.0	1 ()4
C-2	116.4 / shear	118.4 / shear	116.6 / shear	1.02	1.00
C-4	126.3 / shear	128.4 / shear	126.4 / shear	1.02	1.00
C-5	132.4 / shear	156.8 / flexure	132.3 / shear	1.18	1.00

(1): FEM analysis with perfect bond assumption, (2): FEM analysis with interface element

## 5. PARAMETRIC STUDY

To study the effects of various parameters on ultimate shear strength of beams with epoxy bonded continuous steel plates; a parametric study was conducted. The parameters considered are concrete strength, yield strength of steel plate, plate thickness, plate depth to beam web depth ratio, size of beam section and the internal shear reinforcement ratio. The purpose of this study is to find a

suitable modifying factor for the expression of shear contribution of external steel plate to overall shear strength of strengthened beam at ultimate state. Table 2 shows the parameters used in analysis.

Table 2 Parameters considered in parametric study

		nsidered in parametric s	study	
No.	Parameters	Range of values	No. of cases	
1	Concrete strength 'fc' (MPa)	25-70 4		
2	Yield strength of plate 'fy' (MPa)	235-500 3		
3	Plate thick./beam width 'tp/b'	0.0046-0.27		
4	Plate depth/beam depth 'hp/h'	0.25-1.0 4		
5	Shear reinforcement ratio 'p <sub>v</sub> ' (%) +	0.2-0.8	4	
6	Beam size (bxh) (mmxmm)	150x200-1000x2000	4	
	+ $p_v = (A_{sv}/d S_v)*100$	Analyzed cas	ses: 554	
1200	—————————————————————————————————————	1600 tp= 2.3 mm		
	→ tp= 10.0 mm — tp= 20.0 mm	1400 tp= 4.5 mm		
1000	×	— <u>A</u> tp= 6.0 mm		
800		1200 tp= 10.0 m x tp= 20.0 m		
800		£ 1000 + TP= 20.0 m		
		<b>夏</b> 800 十		
600 -	***	S 600		
400 -		(X) 1200		
400	Size 300mmx600mm, As v=0.0, hp/h=0.5	000		
200	0.22 000mm,7.54=0.0, npm=0.0	Size 300mmx600mm, Asv=0.0, fc'=35 MPa		
	20 30 40 50 60 70 80	0.20 0.40	0.60 0.80 1.00 1.20	
	Concrete strength (MPa)		hp/h	
Fi	g. 5 Shear strength vs. concrete strength	Fig. 6 Shear s	trength vs. hp/h ratio	
1800	0 — hp/h= 0.25 — hp/h= 0.50	600		
1600	— <u>A</u> hp/h= 0.75 —— hp/h= 1.00	300X600, fc'=3	5 MPa, tp=2.3 mm — hp/h=0.25	
		<u>9</u> 500	— <u>A</u> hp/h=0.50	
Shear strength (KN) 1000 800 800 800		<u>e</u> 400	→ hp/h=0.75 → hp/h=1.00	
E 1000		Jo		
e 1000	1	· S 300	To the second se	
800		Q 1200		
600	8	000	A	
n 400		Shear contribution of plate (xX)  200  200  200  200  200  200  200  2	A	
	0   Size 300mmx600mm, Asv=0.0,fc'=70 MPa	She	0	
200				
	0 5 10 15 20 25	0.0	0.2 0.4 0.6 0.8	

Fig. 7 Shear strength vs. plate thickness

Fig. 8 Shear taken by plate vs. shear reinf. ratio

Since the concrete contribution to shear strength is a direct function of concrete strength, as concrete strength becomes higher, the shear strength also increases. This relationship between the shear strength versus concrete strength is shown for typical case in Fig. 5. It is found that for thicker steel plates, the strength increases more sharply. Plate depth/beam depth (hp/h) ratio is the most important parameter in shear strengthening technique with continuous steel plates. The larger this ratio, the higher is the contribution of steel plate to shear strength of beam thereby giving higher ultimate shear strength. Obviously, the deeper plates provide larger direct bearing area for shear stress as well as the better confinement to concrete. The effect of this factor on shear strength of beams is shown in Fig. 6. The effect of plate thickness on shear strength of beams is shown in Fig. 7. It is seen that the shear strength increases with plate thickness. The slopes of the curves suggest that for higher hp/h ratios, the increment is higher. From the experiments, it is observed that the

increase in shear strength of beams without shear reinforcement is higher than the beams with shear reinforcement. When steel plate contribution is separated from the nominal shear strength by subtracting the concrete contribution and shear reinforcement contribution, it is found that the plate's contribution to shear strength is higher in beams without shear reinforcement than the beams with shear reinforcement. Fig. 8 shows the internal shear reinforcement ratio versus the shear force taken by steel plate for typical cases. It is found that the plate's contribution to the total shear strength decreases with the increasing shear reinforcement ratio. Maximum plate contribution is obtained for the beams without any internal shear reinforcement.

## 6. SHEAR STRENGTH OF PLATE BONDED RC BEAMS

The ultimate shear strength of RC beams with epoxy bonded continuous steel plates can be computed by adding the individual contributions from concrete, internal shear reinforcement and the external steel plates as shown in Eq. 1.

$$V_p = V_c + V_s + V_p \tag{1}$$

The contribution of each of concrete  $(V_c)$  and internal shear reinforcement  $(V_s)$  are computed following the equation by Okamura and Higai [6] and employing truss analogy, respectively. The expression for shear contribution of externally bonded continuous steel plates ( $V_P$ ) is given by summing up the shear stresses in steel plates over its depth and thickness given by Eq. 2.

$$V_P = \tau_{avg} h_P t_P \tag{2}$$

For the quadratic distribution of shear stress over the plate depth, average shear stress (  $au_{avg.}$ ) is 2/3 of maximum stress ( $au_{max.}$ ). Further, the maximum shear stress is related to uniaxial yield strength  $(f_y)$  by Tresca's yield criterion and after simplification, we get Eq. 3.

$$V_P = \frac{1}{3} f_y h_P t_P \tag{3}$$

However, the steel plates are not fully anchored in concrete, so they are unable to develop their full yielding stress across their depth. Further, to take into account of possible slip between concrete and steel plate, which leads to local debonding; a modifying factor  $\kappa$  is introduced into Eq. 3. The final expression for shear contribution of steel plate at ultimate state takes the form given by Eq. 4.

$$V_P = \frac{1}{3} \kappa f_y h_P t_P \tag{4}$$

From the multiple regression of 554 data sets obtained from parametric analytical study, value of  $\kappa$  is given by Eq. 5.

$$\kappa = 0.68 - 0.27 \, p_v + 0.28 \left(\frac{h_P}{h}\right) - 1.95 \left(\frac{t_P}{b}\right) - 0.007 \left(\frac{f_y}{f_c}\right) \tag{5}$$

where,  $\kappa = \text{modifying factor}$ 

 $t_P$  = thickness of plates both sides of web

 $p_{y}$  = shear reinforcement ratio

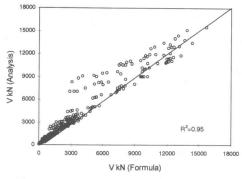
b = width of beam web

 $f_{y}$  = yield stress of steel plates

 $h_P$  = depth of steel plates

h = section depth of beam

 $f_{c}$  = concrete compressive strength



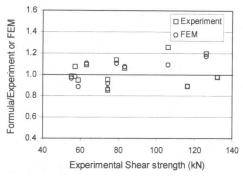


Fig. 9 Correlation between formula and analysis

Fig. 10 Accuracy of proposed formula

Fig. 9 shows the correlation graph between the shear strength of beams by proposed formula and from the parametric study. It is seen that the proposed formula predicts the ultimate shear strengths of beams quite satisfactorily. Fig. 10 shows the normalized graph for shear strengths predicted from the proposed formula and the experiments along with FEM analysis. It is seen that the proposed formula estimates the shear strengths of beams with reasonable agreement being within 20% limit except one particular beam.

## 7. CONCLUSIONS

A nonlinear FEM model is used to analyze the RC beams with web bonded continuous steel plates. Parametric study is conducted to study the effects of parameters such as plate depth to beam depth ratio, plate thickness, concrete strength and internal shear reinforcement ratio. It is seen that the developed FEM model can mimic the overall behavior of beams very well. It is found that the plate depth/beam depth ratio is the most important factor defining the shear strength increment in plated beams. The shear strength increases with concrete strength as well as with the higher hp/h ratio and plate thickness. The contribution of steel plates to the shear strength of beams is higher for the lower internal shear reinforcement ratio. A formula to compute the shear strength of plate bonded beams is suggested. It is found that the proposed formula predicts the ultimate shear strengths of beams quite satisfactorily, thus can be used for practical field applications.

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