論文 Modified Formulation of Cable Strain in Analysis of Externally PC Beam

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ABSTRACT: It is obviously shown that cable strain depends not only on the overall deformation of the beam but also on friction at deviators, at which frictional resistance always exists. In previous studies, friction at deviator was incorporated in calculation of the cable strain in terms of friction coefficients. In this study, formulation of the cable strain is modified in attempt to understand the structural behavior of externally prestressed concrete beam. Criteria of cable slip are also presented. The analytical result in comparison with the test data is presented and a good agreement is found.
KEYWORDS: External cable, PC beam, deviators, displacement, cable strain, slip.

1. INTRODUCTION OF EXTERNALLY PC BEAM

In an externally PC system, when the beam is subjected to bending, the external cable deflection does not follow the beam deflection except at deviator points. As a result, the cable strain cannot be determined from the local strain compatibility between concrete and cable. For the calculation of cable strain, it is necessary to formulate the global deformation compatibility between the end anchorages of the cable. This means that the stress change in the cable is member dependent and is influenced by the initial cable profile, span to depth ratio, deflected shape of the structure, friction at deviators etc. Normally, there is frictional resistance between the cable and the deviator and the cable strain depends on the coefficient of friction. In many studies while calculating the cable strain, two extreme cases are usually considered namely, free slip (no friction) and perfectly fixed (no movement) at deviators. In the first case, cable moves freely throughout the deviators without any restraint and cable is treated as the unbonded internal cable. The cable strain is constant over its entire length regardless of friction at the deviators. Cable strain can be expressed as

$$\Delta\varepsilon_s = \frac{1}{l}\int_0^l \Delta\varepsilon_{cs}\,dx \tag{1}$$

where, $\Delta\varepsilon_s$ and $\Delta\varepsilon_{cs}$ are cable strain and concrete strain at cable level, respectively; l is the total length of the cable between the extreme ends. In the second case, cable is considered perfectly fixed at deviators, meaning that cable strain variation for each segment is independent from the others. The increment of cable strain depends only on the deformation of two successive deviators or anchorages, at which cable is attached to. The strain variation can be expressed as

$$\Delta\varepsilon_{si} = \frac{\Delta l_i}{l_i} \tag{2}$$

where, Δl_i and l_i are the elongated and original length of considered cable segment, respectively. For the former case, if frictional resistance at deviators is neglected, deflection and cracking may be overestimated at the service loading range, whereas for the latter case, if perfectly fixed is assumed,

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the ultimate load capacity may be overestimated. This phenomenon can be seen in **Fig.1** by showing the effect of bond condition of cable at deviator in the analysis of three cases (free slip, slip with friction 0.2 and perfectly fixed), which was calculated by A.M'rad [4]

When friction coefficients at the deviators are considered [2], cable strains on both sides of deviator are different by portion, which is usually caused by friction force. This difference in strain can be expressed in terms of friction coefficient, k_{Di} as

$$\Delta\varepsilon_{s(i+1)} - \Delta\varepsilon_{s(i)} = \frac{k_{Di}}{l_i + l_{i+1}} \int_0^{l_i+l_{i+1}} \Delta\varepsilon_{cs} dx \qquad (3)$$

where, $\Delta\varepsilon_{(si)}$ and $\Delta\varepsilon_{(si+1)}$ are the increment of strain of *(i)* and *(i+1)* cable elements, respectively; $\Delta\varepsilon_{cs}$ is the increment of strain of concrete element at the cable level; l_i, l_{i+1} are the length of *(i)* and *(i+1)* cable elements, respectively.

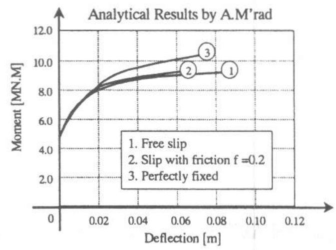

Fig.1 Effect of friction at deviator

In **Eq.(3),** friction coefficients k_{Di} are assumed to be the function of inclination angle of cable having values between 0 and 1.0. Although relationship between the friction coefficient and the angle of cable was proposed in [2], based on intensive analysis of the three examples with different shape of cross section and arbitrary loading scheme. However, these values are not constant during the loading step and should be changed and depended on the loading condition. For beam with having many deviators or multiple span continuous beams, the value and sign of these coefficients are often arisen in calculation and computing process should be repeated until obtaining desirable result. To overcome these difficulties formulation of the cable strain based on the force equilibrium condition at the deviator is modified and will be presented hereinafter.

2. MODIFIED FORMULATION OF CABLE STRAIN

2.1. Force equilibrium at deviator

Fig.2 shows that F_i, F_{i+1} are tensile forces in cable segments *(i)* and *(i+1)* at deviator, correspondingly θ_i, θ_{i+1} are cable angles, respectively. Thus, the force equilibrium condition on *X* direction can be expressed as

$$F_i \cos\theta_i + (-1)^k \mu(F_i \sin\theta_i + F_{i+1} \sin\theta_i) = F_{i+1} \cos\theta_{i+1} \qquad (4)$$

Where $k = \begin{cases} 1 & if \quad F_i \cos\theta_i > F_{i+1}\cos\theta_{i+1} \\ 2 & if \quad F_i \cos\theta_i < F_{i+1}\cos\theta_{i+1} \end{cases}$

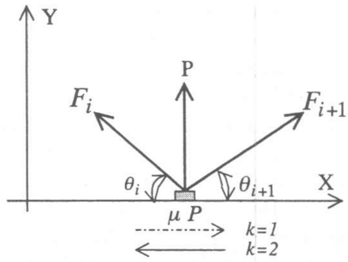

Fig.2. Force equilibrium at deviator

and μ is friction coefficient at the deviator and assumed to be known at each deviator.

To divide both sides of **Eq.(4)** by $E_{ps}A_{ps}$, thus the force equilibrium condition can be expressed in terms of cable strain

$$\varepsilon_{si} \cos\theta_i + (-1)^k \mu(\varepsilon_i \sin\theta_i + \varepsilon_{i+1} \sin\theta_{i+1}) = \varepsilon_{s(i+1)} \cos\theta_{i+1}$$

or

$$\left[\cos\theta_i + (-1)^k \mu \sin\theta_i\right]\varepsilon_{si} + \left[-\cos\theta_{i+1} + (-1)^k \mu \sin\theta_{i+1}\right]\varepsilon_{s(i+1)} = 0 \qquad (5)$$

where, E_{ps} and A_{ps} are the elastic modulus and area of prestressing cable; $\varepsilon_{si}, \varepsilon_{s(i+1)}$ are the cable strains at both sides of the deviator, respectively.

2.2. Incorporation of cable strain in matrix form

Like the previous formulation, the total deformation of cable element should be equal to the total deformation of concrete element between end anchorages.

$$\sum_{i=1}^{n} l_i \Delta\varepsilon_{si} = \int_o^l \Delta\varepsilon_{cs} dx \qquad (6)$$

where, $\Delta\varepsilon_{si}$ and l_i are the cable strain and the cable length of considering segment; $\Delta\varepsilon_{cs}$ is the concrete strain at the cable level; l is the total length of the cable between the extreme ends.

From **Eq.(5)** and **Eq.(6),** the cable strain can be incorporated in matrix form

$$\begin{bmatrix} l_1 & l_2 & l_3 & \cdots\cdots & l_{n-1} & l_n \\ C_1+(-1)^k\mu S_1 & -C_2+(-1)^k\mu S_2 & 0 & \cdots\cdots & 0 & 0 \\ 0 & C_2+(-1)^k\mu S_2 & -C_3+(-1)^k\mu S_3 & \cdots\cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots\cdots & -C_{n-1}+(-1)^k\mu S_{n-1} & 0 \\ 0 & 0 & 0 & \cdots\cdots & C_{n-1}+(-1)^k\mu S_{n-1} & -C_n+(-1)^k\mu S_n \end{bmatrix} \begin{Bmatrix} \Delta\varepsilon_{s1} \\ \Delta\varepsilon_{s2} \\ \Delta\varepsilon_{s3} \\ \vdots \\ \vdots \\ \Delta\varepsilon_{s(n-1)} \\ \Delta\varepsilon_{sn} \end{Bmatrix} = \begin{bmatrix} \int_0^l \Delta\varepsilon_{cs}dx \\ 0 \\ 0 \\ \vdots \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

or $$[M]\{\Delta\varepsilon_s\}=[N]\{d\} \quad \Rightarrow \quad \{\Delta\varepsilon_s\}=[M]^{-1}[N]\{d\} \tag{7}$$

where, C and S are denoted as cosine and sine of the cable angle, subscript under these letters is indicated the cable angle number; $\{d\}$ is the nodal displacement vector.

3. EVALUATION OF CABLE SLIP AT DEVIATOR

3.1. Criteria of cable slip at deviator

The most important thing is that cable slip is not reversible. It means that once cable slip occurs at deviator *(i)*, for example from the deviator *(i)* toward the deviator *(i+1)*, the slip will continuously occur in this direction until no occurrence of the slip at this deviator. Change in direction of the cable slip will never take place.

From **Eq.(4),** the force equilibrium condition can be rewritten again : $F_{dv} = F_{fr}$ (8)

where $F_{dv} = F_{i+1}\cos\theta_{i+1} - F_i\cos\theta_i$ is called as driving force

$F_{fr} = (-1)^k \mu P = (-1)^k \mu(F_i \sin\theta_i + F_{i+1}\sin\theta_{i+1})$ is called as friction force

If the driving force is not equal to zero ($F_{dv} \neq 0$), then **Eq.(8)** can be written as : $\frac{F_{fr}}{F_{dv}} = \lambda = 1$ (9)

Now, at any loading stage, there are three possibilities

1. If $\lambda > 1$ - The friction force is greater than the driving force. *Slip can not occur*
2. If $\lambda = 1$ - The friction force is equal to the driving force. *The force equilibrium condition*
3. If $\lambda < 1$ - The friction force is less than the driving force. *Slip must occur*

From above condition, at any deviator, the cable slip will occur only if the friction force is less than the driving force. Here should pay attention that the friction force and the driving force should have the absolute value.

3.2. Calculation of cable slip

At certain loading stage, when slip occurs at the deviator i.e. $\lambda < 1$. In this case, slip occurs and will continuously occur until the equilibrium condition at the deviator after slip is achieved. The redistribution of the cable forces at both side of the deviator is allowed though slippage. An amount of slip at one deviator depends not only on an additional force of this deviator, but also on additional forces of two adjacent deviators i.e. one is from the left side and the other is from the right side. If F_i, F_{i+1} and $F_i^{'}, F_{i+1}^{'}$ are denoted as the cable forces at either side of the deviator just before and after slip, respectively. Then, the cable force after slip is assumed to be equal to sum of the cable force before the slip plus or minus the additional forces, ΔF_{ad} which is caused by slip at deviators and these can be expressed

$$\begin{aligned} F_i^{'} &= F_i + (-1)^{k(i)}\Delta F_{ad(i)} + (-1)^{k(i-1)}\Delta F_{ad(i-1)} \\ F_{i+1}^{'} &= F_{i+1} + (-1)^{k(i)+1}\Delta F_{ad(i)} + (-1)^{k(i+1)}\Delta F_{ad(i+1)} \end{aligned} \tag{10}$$

The sign of the additional force depends on the slipping direction i.e. depends on coefficient k, which is defined in **Eq.(4)**. After the slip, the force equilibrium condition should be satisfied and again expressed as

$$F_i^{'}\cos\theta_i + (-1)^{k(i)}\mu\left(F_i^{'}\sin\theta_i + F_{i+1}^{'}\sin\theta_{i+1}\right) = F_{i+1}^{'}\cos\theta_{i+1} \quad (11)$$

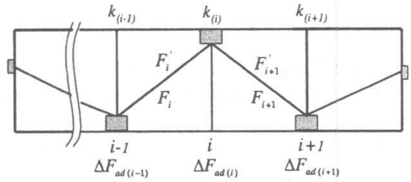

Fig.3 Equilibrium condition at slipping zone

Substituting **Eq.(10)** into **Eq.(11)**, then after some manipulation, we obtain

$$A_i.\Delta F_{ad(i-1)} + B_i.\Delta F_{ad(i)} + C_i.\Delta F_{ad(i+1)} = F_{dv(i)} - F_{fr(i)} \quad (12)$$

where, $A_i = \left[(-1)^{k(i-1)}\left(\cos\theta_i + (-1)^{k(i)}\mu\sin\theta_i\right)\right]$; $\quad C_i = \left[(-1)^{k(i+1)}\left(-\cos\theta_{i+1} + (-1)^{k(i)}\mu\sin\theta_{i+1}\right)\right]$

$$B_i = \left[\mu\left(\sin\theta_i - \sin\theta_{i+1}\right) + (-1)^{k(i)}\left(\cos\theta_i + \cos\theta_{i+1}\right)\right];$$

For a special case, when cable can not slip at two adjacent deviators, i.e. the additional forces at these deviators are equal to zero. Hence , **Eq.(12)** can be rewritten as

$$B_i.\Delta F_{ad(i)} = F_{dv(i)} - F_{fr(i)} \quad (13)$$

It is assumed that cable can not slip at two extreme deviators of the slipping zone, therefore, two cases are possible. If cable did not slip at the left deviator, then $\Delta F_{ad(i-1)} = 0$, whereas, if cable did not slip at the right deviator, then $\Delta F_{ad(i+1)} = 0$. Therefore, **Eq.(12)** can be rewritten for two cases in **Eq.(14)** and **Eq.(15)**, respectively.

$$B_i.\Delta F_{ad(i)} + C_i.\Delta F_{ad(i+1)} = F_{dv(i)} - F_{fr(i)} \quad (14)$$

$$A_i.\Delta F_{ad(i-1)} + B_i.\Delta F_{ad(i)} = F_{dv(i)} - F_{fr(i)} \quad (15)$$

To satisfy the global equilibrium condition at slipping zone, then the additional forces at deviators are defined by incorporating **Eq.(12)**, **Eq.(14)** and **Eq.(15)** in a general matrix form

$$\begin{bmatrix} B_1 & C_1 & 0 & \cdots & 0 & 0 & 0 \\ A_2 & B_2 & C_2 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & A_{(n-1)} & B_{(n-1)} & C_{(n-1)} \\ 0 & 0 & 0 & \cdots & 0 & A_{(n)} & B_{(n)} \end{bmatrix} \begin{Bmatrix} \Delta F_{ad(1)} \\ \Delta F_{ad(2)} \\ \vdots \\ \Delta F_{ad(n-1)} \\ \Delta F_{ad(n)} \end{Bmatrix} = \begin{Bmatrix} F_{dv(1)} - F_{fr(1)} \\ F_{dv(2)} - F_{fr(2)} \\ \vdots \\ F_{dv(n-1)} - F_{fr(n-1)} \\ F_{dv(n)} - F_{fr(n)} \end{Bmatrix}$$

or $$[S]\{\Delta F_{ad}\} = \{T\} \quad \Rightarrow \quad \{\Delta F_{ad}\} = [S]^{-1}\{T\} \quad (16)$$

Therefore, an amount of slip at deviator can be determined as

$$Slip_i = \frac{\Delta F_{ad(i)}}{A_{ps}E_{ps}} l_i \quad for \quad k = 1 \quad \text{Cable slip is from right to left} \quad (17)$$

$$Slip_i = \frac{\Delta F_{ad(i)}}{A_{ps}E_{ps}} l_{i+1} \quad for \quad k = 2 \quad \text{Cable slip is from left to right} \quad (18)$$

where, E_{ps} and A_{ps} are the elastic modulus and area of prestressing cable; l_i, l_{i+1} are the cable length of segments *(i)* and *(i+1)*, respectively.

Therefore, the possibility for a slip at any deviator depends on the inclination angle of cable, force difference in cable and friction coefficient. For every loading step, according to the deformation shape of the beam, the inclination angle of cable will change. As a result of the deflected shape of the beam, the change of cable angle should be considered in calculation of cable slip.

4. NUMERICAL ANALYSIS

To demonstrate the applicability of the modified formulation and also in comparison with experimental data for the cable slip, the full scale test of precast segmental bridge with external cable which was carried out at a project of expressways in Bangkok [3] is taken to analyze as a numerical example. Hereinafter, the analytical results in comparison with experimental data will be presented.

4.1. Introduction of analytical model

The analytical example is carried out for simple span with 43.25m in clear span length. The test beam has box cross section with 2.4m in height and prestressed by 6 cables of type 19K15 and 12K15 at either side of cross section. Cables were stressed about 70% of the ultimate strength of cable. Three deviators are provided at distance as shown in the layout scheme of the test beam (see **Fig.4**) and material properties is in **Table 1**. It is assumed friction coefficients at all deviators are equal to 0.2 and stress – strain relations for concrete and cable are shown in **Fig.5**

Table 1. Material properties Unit : N/mm^2

Concrete		Steel		Prestressed cables		
σ_c	Es	σ_{sy}	E_s	σ_{py}	σ_{pu}	E_p
57.0	4.3x10^4	390	2.1x10^5	1600	1920	1.93x10^5

4.2. Discussion of analytical results

Fig.6 shows load-displacement relationship of midspan section for analytical model as well as results of the unbonded model calculated by Dan Tassin[1]. The maximum superimposed moment is about 58150kN.m, correspondingly the maximum deflection is about 34.4cm. The predicted load-displacement response has very good agreement with the test data. In comparison with other calculation, the analytical model gives better results.

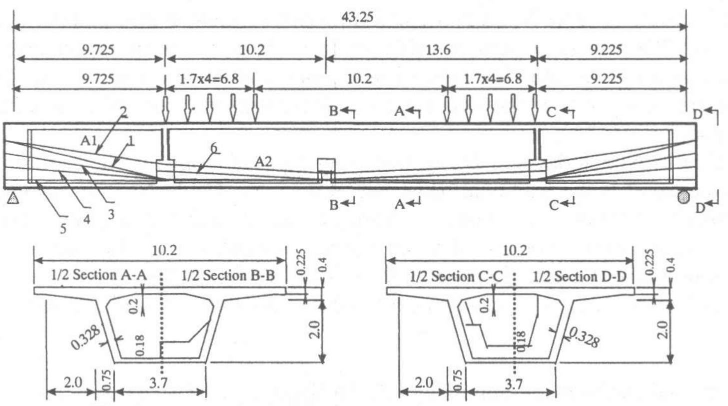

Fig.4 Layout scheme of test beam (All dimensions in m)

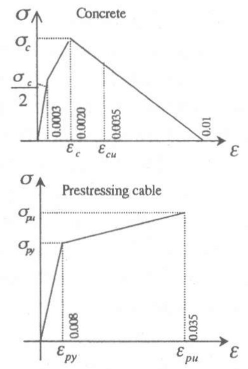

Fig.5 Stress – Strain relationships

Fig.7 shows concrete strain at the midspan section against the superimposed moment. From this figure, it could be seen that the concrete strain curve is nearly the same as the experiment. At the ultimate stage, increment of the concrete strain is about 0.00133, while experimental value is about 0.0012.Value of the concrete strain of the unbonded model predicted by Dan Tassin is not available, so comparison could not be made.

Fig.6 Load-Displacement relationship

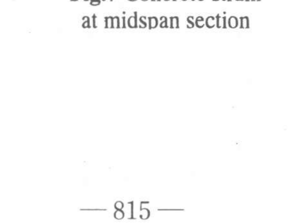

Fig.7 Concrete strain at midspan section

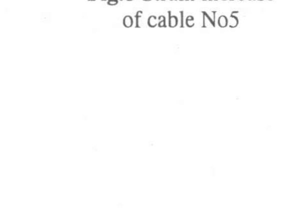

Fig.8 Strain increase of cable No5

Increment of cable strain for cables No 5,No6 and No1 and results calculated by Dan Tassin are presented in figures from **Fig.8** to **Fig.10**, respectively. Almost the strain behaviors of all cables are characterized by linear and nonlinear portion with departure from linear at a superimposed loading about 38.000 kN.m. Among 6 cables from No1 to No6, cable No5 has the smallest inclination angle, so increment of cable strain at the end portion and the midspan portion is not so much different, almost the same. This phenomenon could be explained that with the smaller deviation angle and thus the smaller frictional force, the strain variation may be redistributed throughout slippage.

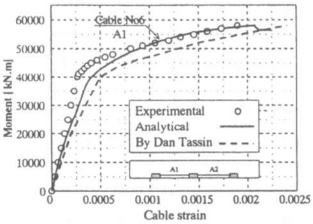

Fig.9 Strain increase of cable No6

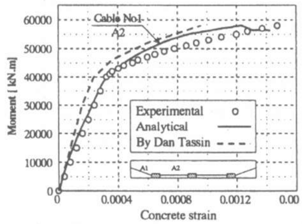

Fig.10 Strain increase of cable No1

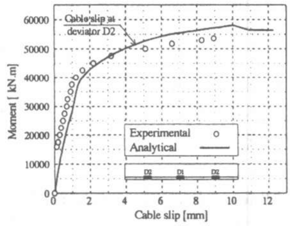

Fig.11 Slip of cable No5 at deviator D2

Cable No6 is located at midspan portion and has only one deviation point as shown in **Fig.9** Like cable No5, cable No6 has small inclination angle and cable strain of two portions is not significantly different. Because of the short length, greater strain increase is found in cable No6 compared to others. Maximum cable strain is about 0.0021 for the analytical model.

Fig.10 shows strain increase of cable No1 against superimposed moment at midspan portion. It could be seen that up to 40000kN.m increment of cable strain is the same as in experimental observation. However, after that for given superimposed moment, increase of cable strain is smaller in comparison with the experimental data. At the ultimate stage, increment of cable strain is about 0.00124 for the analytical model.

Fig.11 shows slip of cable No5 at deviator D2 against superimposed moment. The cable slip occurs, once the driving force exceeds the frictional resistance at deviator. Cable No5 has smaller deviator angle and hence smaller frictional resistance resulting in slip at each loading step. This reason could be explained that the strain increase of every portion is mostly the same and strain redistribution is obviously took place through slippage as mentioned above. At the ultimate stage, cable slip at deviator is about 10mm and in comparison with test data a good agreement is found.

5. CONCLUSIONS

The following conclusions can be made from this study. By applying modified formulation of cable strain, the structural behavior of externally PC beam up to the ultimate state can be satisfactorily predicted and cable slip at each loading step is also investigated. The results show good accuracy. In comparison with other calculation, a better result is found from analytical model.

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