# 論文 Cyclic Behavior of the RC Columns Using the Modified Lattice Model

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**ABSTRACT:** The Modified Lattice Model is extended into the three dimensions. Cyclic torsion is selected to study the applicability of the suggested model. Investigation of the response of reinforced concrete columns under the cyclic torsion is studied using the suggested 3-D Model. Evaluation of shear strength of the existing column is carried out. The response of different RC columns is predicted. The change of the width of the arch element is shown during the loading stages for the studied cases. The numerical results for both of the Normal Lattice Model and the Modified Lattice Model are compared successfully with the experimental results.
**Key words:** Modified Lattice Model, Normal Lattice Model, total potential energy, and cyclic torsion.

## 1. INTRODUCTION

Cyclic loading is commonly applied for the RC structures. However, the applied loads are generally applied in three dimensions. Analysis of RC structures in 3-D is quite difficult. That is why a few researches only are interested in such kind of numerical analysis. Even the experimental work in three dimensions is so limited.

In this paper, the Modified Lattice Model with its suggested technique by the authors is extended into the three dimensions. The cyclic behavior of the RC columns is suggested as an example to check the applicability of the suggested model in 3-D. An investigation of the response of reinforced concrete columns under the reversed cyclic torsion is studied. Evaluation of the shear strength for several existing columns is carried out. The response of different RC columns is shown comparing with the experimental results. For the numerical results using 3-D Modified Lattice Model, and using the new technique to calculate the width of the arch element inside each step of the calculation, the change of arch element along all the loading stages is predicted. The numerical results using both of Normal lattice Model and the Modified Lattice Model are also shown together to capture the accuracy of the calculation in between. The numerical results will be shown with the experimental results for the comparisons.

## 2. CONFIGURATION OF THE 3-D MODIFIED LATTICE MODEL UNDER CYCLIC TORSION

**Fig. 1** shows the configuration of a reinforced concrete column simulated into 3-D Modified Lattice Model. In case of 3-D Modified lattice Model, the column is represented by four simple truss planes, which are orthogonal to each other. Each truss plane follows all the assumptions of the Modified lattice Model in 2-D [2]. For the trusses in the wider face of the column, the inclination angle of the diagonal members is fixed at 45 degrees. For the other two sides in the short direction, the position of the nodes are kept exactly as they are determined from the wider faces. So, in the short side of the cross-section, the diagonal angles will usually less than 45 degrees. In case of reinforced concrete column under the cyclic torsion, which is considered as an example to study this model, all

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the corners of the column is under tension stresses. Area of the vertical strut equals to the corner area, which is bisected by the equivalent wall thickness [3]. In **Fig. 1** the solid lines representing the arch elements. The sub-diagonal members are kept as two couples only in the direction of the depth. The thickness of the arch element is changing continuously according to the minimum total potential energy inside each step of the calculation [2]. **Fig.2** shows the cross-section of the 3-D Modified Lattice Model, considering the tension zone area at each corner zone, with an area equals to $t^2_w$ and $t_w$ as in **Eq. 1** [3].

$$t_w = 1.2 * A_c / P_c \qquad (1)$$

where $t_w$ is the wall thickness of the hollow section, Ac is the area of the solid section and equals to b*d and Pc is the outer perimeter of the solid cross-section

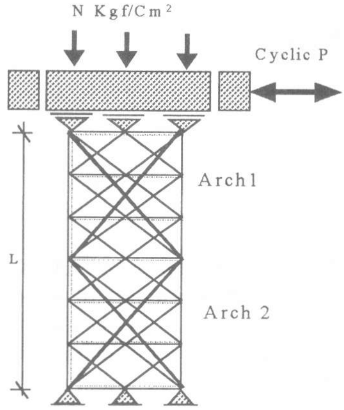



**Fig.1** 3-D Lattice Model for Reinforced Concrete Column

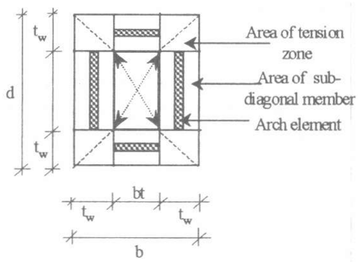



**Fig.2** Cross-section of 3-D Lattice Model

## 3. MATERIAL MODELS IN TWO DIMENSIONAL PLANE STRESS CONDITION

Constitutive equations for the used concrete and steel have a major effect on the output results under the reversed cyclic load. For example cyclic strength and stiffness degradation may prove un-influential on the final results in cases where the phenomena of extensive cracking and the yielding of steel dominate the overall behavior.

### 3.1 Uniaxial Concrete Constitutive Equation in Tension

The diagonal tension member of concrete resists the principle tensile stress resulting from shear force. The model, which is suggested by Rots et al [4], is considered in this study as shown in **Fig. 3**. It connects the points on the envelope where unloading starts straight with the origin of the coordinate system. Unloading and reloading run on the same line, i.e. there is no hysteresis loop. After running back and forth the envelope function is valid again. This and similar models approximate the real behavior relevantly in the overall behavior. The equations for this model are shown as in **Eq. (2)** and **Eq. (3)**.

For ascending branch $(\varepsilon_r < \varepsilon_{cr})$ $\qquad \sigma_r = E_c \varepsilon_r \qquad (2)$

For descending branch $(\varepsilon_r \geq \varepsilon_{cr})$

$$\sigma_r = (1-\alpha) f_t \exp\left[-m^2\left(\frac{\varepsilon_r}{\varepsilon_{cr}}-1\right)^2\right] + \alpha f_t \qquad (3)$$

where $\varepsilon_r$ and $\sigma_r$ are the strain and the stress of the tension element, respectively. $\varepsilon_{cr}$ is the strain at the cracking of concrete and $E_c$ is the modulus of elasticity of concrete. The stress-strain behavior of concrete in tension is elastic before cracking and exhibits softening after cracking. The softening slope should take care of fracture energy for plain concrete and tension-stiffening effect for reinforced concrete [2]. **Eq. (2)** shows the elastic behavior before cracking. In **Eq. (3),** *m* can be varied to simulate the appropriate fracture energy for plain concrete. Appropriate $\alpha$ can be chosen to simulate the residual stress in the final stage of damage for simulating tension-stiffening effect in reinforced concrete. In this research $m=0.5$ and $\alpha=zero$ are adopted. To control the constant fracture energy, The One-Forth Model of Tension Softening Curve is considered as it is shown in **Fig. 4**. It is elastic before cracking, however, once crack occurs, concrete exhibits the tension-softening behavior. Therefore, after cracking, the tension-softening curve for concrete is applied. Employed softening curve is the one-forth model as in **Fig. 4**. The crack width, w, shown in **Fig. 4** is divided by the length of the diagonal tension member of concrete and converted into the strain. The fracture energy for concrete in this study is kept constant and adopted from the recommendation by RILEM report 1991 [4].

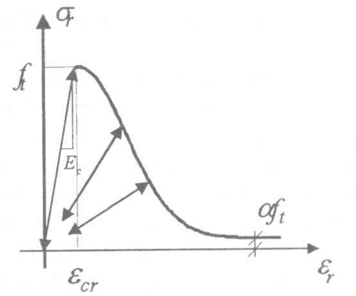


**Fig.3** Tensile Stress-Strain Curve of Concrete

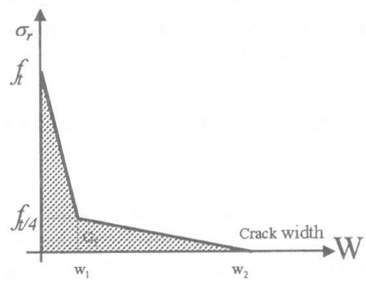


**Fig. 4** The One-Forth Model of Tension-Softening Curve

### 3.2 Uniaxial Concrete Constitutive Equation in Compression

To consider the compression-softening behavior of crushed concrete, the model proposed by Collins et al. [5] is adopted. In that model the softening coefficient was proposed as a function of the transverse tensile strain. So, the tension and compression members are considered as a pair together. **Eq. (4)** shows the compressive stress-strain relationship of concrete in this study.

$$\sigma_c = -\eta f_c' \left( 2\left(\frac{\varepsilon_c}{\varepsilon_o}\right) - \left(\frac{\varepsilon_c}{\varepsilon_o}\right)^2 \right) \tag{4}$$

where, the peak softening coefficient $$\eta = \frac{1.0}{0.8 - 0.34\left(\frac{\varepsilon_r}{\varepsilon_o}\right)} \leq 1.0 \tag{5}$$

In Eq. 5, $\varepsilon_r$ and $\varepsilon_c$ respectively are the strain in the tension and compression elements for each pair of subdiagonal members. $\varepsilon_o$ is the maximum strain for the member without softening consideration. The stress-strain for the compression concrete elements is shown in **Fig.5**. In this model and in case if unloading happens at the point A, the unloading direction will follow the same slope of the initial tangential at the origin point down to the point B. At any point in the direction from A to B, if the reloading started to happen again, it will follow the same line but in the direction from point B to point A. If it reaches up to point B, the strain will be recalculated to determine point C. From point C, the direction of reloading will be in the direction from C to A. During the branch from C-A, if the unloading happens again, it will come back in the same line and in the direction from A to C. From Point A, the calculated value of the stress and strain will follow the curve, which is suggested before by **Eq. (4)**.

### 3.3 Stress-Strain relation for Steel Members

Numerous models of the hysteric stress-strain behavior have been proposed. Most of them are phenomenological in that, they attempt to describe the behavior of the reinforcement. It is of interest to note that, most analytical models proposed to date, have attempted to address the hysteric behavior of reinforcement. But the range of cyclic strain history, to which reinforcing bars are likely to be subjected, differs significantly from that of structural steel members in that compressive strains are not as large as tensile strains. **Fig. 6** shows the selected stress-strain model to represent the constitutive equation for the reinforcement members.

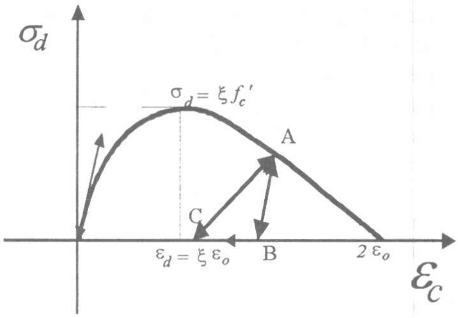



**Fig 5** Compression Stress-Strain Curve For Concrete Element

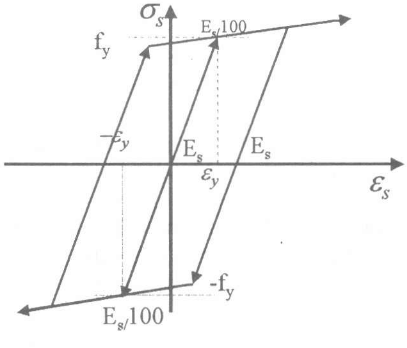
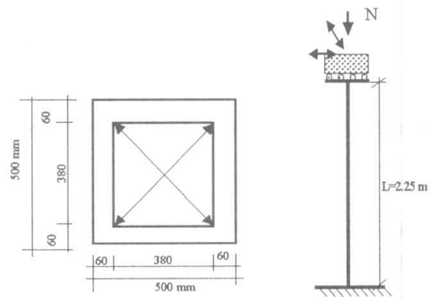



**Fig. 6** Constitutive Model for the Reinforcement Members

## 4. OUTLINE OF THE EXPERIMENTAL DATA

For the case of cyclic torsion several specimens are tested by Bernt Jakobsen et al. [1]. Under the cyclic torsion, the cross-section of all these specimens were as a box-shaped cross-section as it is shown in **Fig.7**. At the top of the column there is an axial load "N" applied for some of these columns to study the effect of the axial load. Three specimens only which are considered as a direct example for the case of pure cyclic torsion are picked up and analyzed numerically by the suggested 3-D Modified lattice Model. $C_4$, $C_6$, and $C_7$ are the selected columns to be analyzed. Specimen $C_4$ was purely twisted, whereas $C_6$ and $C_7$ were subjected to a constant axial force as it is shown in **Table 1**. This force simulated the dead weight of the superstructure. The structural parameter varied between $C_6$ and $C_7$ was the reinforcement content, for the longitudinal reinforcement and for the stirrups. The concrete parameters and the reinforcement details are shown in Table 1.




(a) Cross-Section (b) RC Column under Cyclic Torsion

**Fig. 7** Configuration of the Experiment Test

## 5. ANALYSIS OF THE NUMERICAL RESULTS

### 5.1 General Behavior

**Figures 8**, **9** and **10** show the relations between the torsion moment and angle of rotation for the specimens $C_4$ and $C_6$, respectively. The numerical response and the experimental results are shown in each figure. The numerical results are shown for both of the Modified Lattice Model and the Normal lattice Model. As a general behavior for all the studied specimens, the numerical relation in between the Normal lattice Model and the Modified Lattice Model are kept constant up to the

**Table 1** Layout of the Experimental Data

| Spec-men | $F_c$ MPa | $F_t$ MPa | $E_c*10^{-4}$ MPa | $F_{sv}$ MPa | Main Reinforcement $Cm^2$. | Stirrups S mm | Axial Load KN |
|---|---|---|---|---|---|---|---|
| $C_4$ | 31.9 | 2.6 | 2.64 | 400.0 | 12 | 75 | 0.0 |
| $C_6$ | 31.9 | 2.6 | 2.64 | 400.0 | 20 | 150 | 325 |
| $C_7$ | 31.9 | 2.6 | 2.64 | 400.0 | 12 | 100 | 325 |

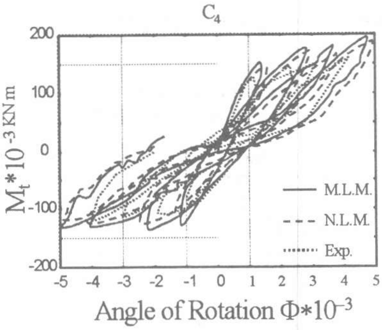



**Fig. 8** Torsional Moment and Angle of Rotation for Column $C_4$

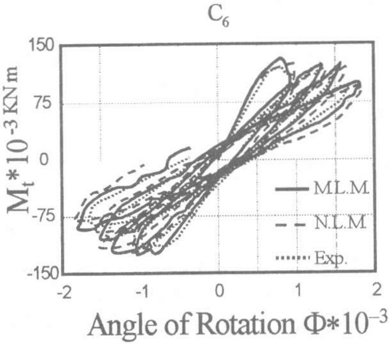



**Fig. 9** Torsional Moment and Angle of Rotation for Column $C_6$

cracking torsion moment. After that the results using the Normal lattice Model are kept lower than that case of the Modified Lattice Model. All the sequence cycles are kept smoothly with a good agreement with the experimental results, except in some cycles only. That is which related to the modeling of the used constitutive equation of the reinforcement, which has to be modified. In case of the numerical results using Modified Lattice Model, the results become closer to the experimental results. In case of $C_4$ which has an axial load "N", the response of the torsion moment and angle of rotation becomes quite larger than the other cases without this axial load as in **Fig.8.** Furthermore, this response increases with the increase of the stirrup amounts inside the columns as shown in **Fig. 9 and Fig.10.**

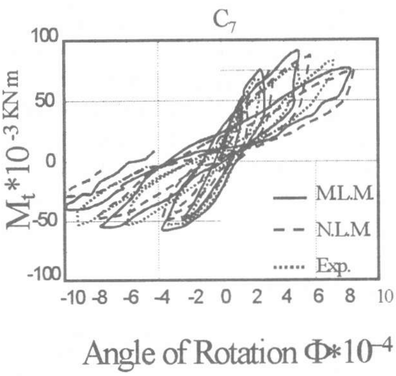



**Fig. 10** Torsional Moment and Angle of Rotation for Column $C_7$

### 5.2 Behavior of the Arch Element

In the Modified Lattice Model, the thickness of the arch element is changed during the calculation up to the final loading stage. The thickness of the arch element is calculated inside each step of the calculation. The value of this thickness is determined by minimizing the total potential energy for all the structure along all the loading stages. But in case of the Normal Lattice Model, the thickness of the arch element is calculated only one time during the elastic stage. The change of the thickness of the arch element for each of the studied columns is predicted. Considering the arch 1 and arch 2 as it is shown in **Fig.1**. In case of column $C_4$, **Fig. 11** and **Fig. 12** show the change of the arch elements, arch 1 and arch 2 respectively. In these two figures and as a general behavior, the rate of increasing the thickness of the arch element in the beginning cycles is less than that case of the final cycles. Even the rate of this changing is increasing gradually from the beginning up to the end. From those figures, the thickness of the arch element is increasing significantly during the loading from the elastic stage up to the final loading. That is to maintain the appropriate response close to the real behavior of the column.

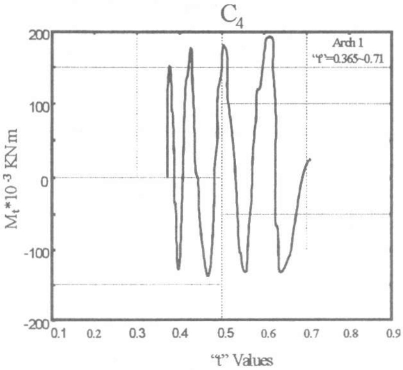



**Fig.10** Behavior of the Arch Element for Column **$C_4$-Arch 1**

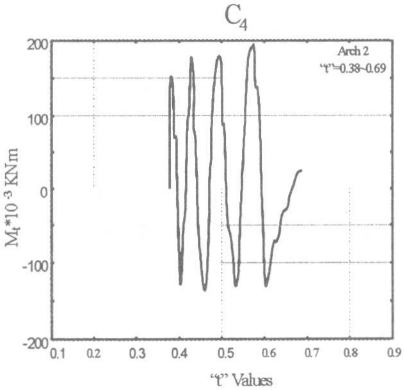



**Fig.11** Behavior of the Arch Element of Column **$C_4$-Arch 2**

## 6. CONCLUSION

In this paper, the suggested 3-D Modified Lattice Model is extended to incorporate the cyclic torsion effect. Both of the numerical results using the Modified Lattice Model and the Normal Lattice Model are shown comparing with the experimental results. In case of using the Modified Lattice Model, the numerical results for both of Modified Lattice Model and Normal lattice Model are kept constant during the elastic stage, after that, the results using the Modified Lattice Model become lower than that case of Normal lattice Model. The change of the width of the arch element is increasing gradually from the elastic stage up to the failure case. The rate of increasing is also increasing gradually from stage to stage. The Modified Lattice Model is closer to the experimental results than the normal Lattice Model to capture the cyclic loading effect.

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