論文 Localization Analysis of Reinforced Concrete Beams Based on Finite Element Method

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ABSTRACT: A method based on finite element method has been used to simulate localization phenomenon in RC elements. This paper is dedicated to the modeling of the shear band localization in context of large strain accumulation in a narrow band without substantially affecting the strain in the surrounding material. This phenomenon frequently occurs accompanying inelastic deformation and material acoustic tensor looses its positive definiteness. Furthermore, finite element method is used to simulate this phenomenon. The model, when used in finite element context, gives mesh-insensitive results regarding to the width of the shear band.
KEYWORDS: localization phenomenon, bifurcation, shear failure, shear band, finite element method, reinforced concrete

1. INTRODUCTION

Mechanism of shear failure in concrete as well as reinforced concrete has been a long-standing key problem that is not fully clarified and argued from various angles. Experiments show that in close vicinity to the peak point, before or after that, a localized damage band could often be observed in reinforced concrete structures. For some kind of structures like shear walls, concrete cracks occur firstly at the early stage of load, however as the external load increased to the certain extent, a damage band would occur within a short interval of time along the direction which is entirely different from the initially formed crack direction. In the case of reinforced concrete beams, regarding to the amount of the longitudinally bars and also web reinforcement, damage band direction usually is near to the initially formed crack direction. Once this phenomenon occurred, the structure would fail with large strain localizing inside the damage band without affecting the other parts of the structure. For these kinds of structures, the post peak behaviors and also failure point of loading and displacement can not be obtained correctly by analysis without knowing what is the reason for this phenomenon and how to simulate this phenomenon by numerical analysis.

To overcome this difficulty we used a special model of shear band localization which the localized zone is embedded within the element. In this method, the width of shear band or localized zone is less than an element size. An approximation of the element size can be calculated by square root of the element area. It is important to note that localization takes place due to strain jump in the shear band and in fact there is no continuity for strain field in the localized element. For this purpose, it is necessary to recalculate stiffness and $\boldsymbol{B}$ matrices for localized element considering strain discontinuity and bifurcation phenomenon. In the discontinuity element, a length scale parameter is introduced which is a material parameter and can be related to the size of the fracture process zone. This solves

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the mesh-size dependence problem as present when a standard crack model is used. To calculate band direction and finite element formulation, we review some aspects of the general theory of strain localization and bifurcation.

2. MECHANISM OF LOCALIZATION AND FINITE ELEMENT FORMULATION

Localization is a phenomenon that large strain accumulates inside a part of material without substantially affecting the strain in the surrounding materials. As it is known that material instability which can be possibly lead to the localization phenomenon in structures, occurs when acoustic tensor loss its positive definiteness. The physical mechanism for this phenomenon is that strain field across the damage band can be possibly takes a jump, while the equilibrium of the stress across the damage band remains to be satisfied (Fig.1). To derive under what condition this kind of localization is triggered, we will adopt the element level bifurcation analysis of Ortiz, *et al.* [1]

The incremental stress-strain relation can be put in the form

$$\dot{\sigma}_{ij} = D_{ijkl}\dot{\varepsilon}_{kl} \quad (1)$$

where D_{ijkl} is the tangential constitutive matrix of the material. Taking jump of (1) gives

$$\|\dot{\sigma}_{ij}\| = D_{ijkl}\|\dot{\varepsilon}_{kl}\| \quad (2)$$

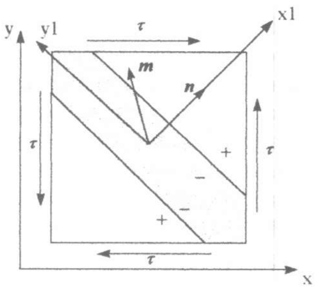

Fig. 1 Localized damage band

Equilibrium across discontinuity planes requires that traction t be continuous then

$$\|\dot{t}_j\| = \|n_i\dot{\sigma}_{ij}\| = 0 \quad (3)$$

where n is the normal to the plane. Combining (2),(3)

$$n_i D_{ijkl}\|\dot{\varepsilon}_{kl}\| = 0 \quad (4)$$

The criteria for this kind of localization phenomenon can be expressed as

$$\det(A(n)) = 0 \quad (5)$$

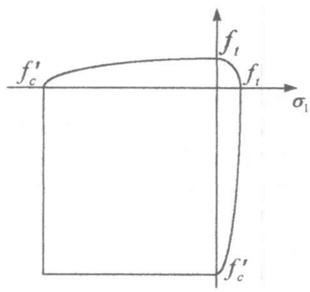

Fig. 2 Failure surface for concrete

where $A(n)m_k = (n_i D_{ijkl} n_l)m_k = 0$ is the acoustic tensor (Fig. 1). Equation 1 can be rewritten as:

$$A_{ij}(n)m_k = (n_i D_{ijkl} n_l)m_k = 0 \quad (6)$$

If the material satisfies Eq.(5) ,then increments of strains can have a jump along the discontinuous surface(Fig.1) and is given by $\|\Delta\varepsilon\| = \Delta\varepsilon^+ - \Delta\varepsilon^- = \alpha\hat{T}$ where in two dimensions

$$\hat{T} = \begin{bmatrix} m_1 n_1 \\ m_2 n_2 \\ m_1 n_2 + m_2 n_1 \end{bmatrix}, \quad \varepsilon = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ 2\varepsilon_{xy} \end{Bmatrix} \quad (7)$$

then

$$\|\Delta\varepsilon\| = \alpha(m_1n_1, m_2n_2, m_1n_2 + m_2n_1)T \tag{8}$$

where α is the strength of the jump mode $\hat{T}$,while the equilibrium across the discontinuous planes remains to be satisfied. In the local coordinate system $x_1 - y_1$, increment of the stress is

$$\|\Delta\sigma\| = (\Delta\sigma_{x1}, \Delta\sigma_{y1}, \Delta\tau_{x1y1})^T = (0, \Delta\sigma_{y1}, 0)^T \tag{9}$$

In order to take a factor of localization into consideration, the finite element with embedded discontinuous displacement field has been used (Belytschko,T. *et al*[2]). In the absence of localization, the strain increment is defined by $\Delta\varepsilon = B\Delta d$, where Δd is the increments of nodal displacement. Once localization occurs, the strain-nodal displacement matrix ***B*** can be calculated as:

$$\overline{B} = \begin{cases} \overline{B}_L = (I + \alpha_P(h/b - 1)TT^T)B \\ \overline{B}_N = (I - \alpha_P TT^T)B \end{cases} \tag{10}$$

where B_L is related to localized zone and B_N is related to nonlocalized zone. T matrix is as $T = \hat{T}/|\hat{T}|$ and the strength of the jump mode $\alpha = \alpha_P(h/b)T^T B\Delta d$ where ***h*** is the square root of the total area of the element (an approximation of the element size) and α_P can be evaluated using Eq.8. For the sake of the limit of this paper, details to determine ***n, m*** and α_P which are complicated, will not be given in this paper (see ref. 2 for more details). ***b*** is the shear bandwidth. Note that the analysis which ***B*** matrix is calculated by Eq.10 is called localized analysis in this paper. Standard FEM is that strain jump is not taken in account and ***B*** matrix is calculated by standard form of calculation without localization effect.

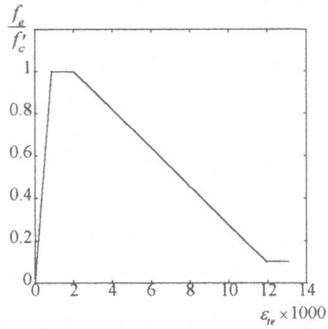

Fig. 3 Uniaxial concrete compression response

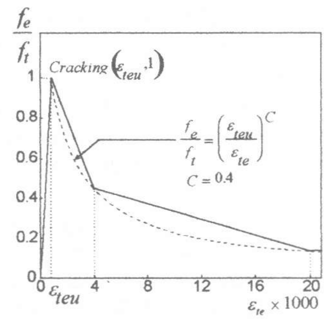

Fig. 4 Uniaxial concrete tensile response and linear approximation

It is notable that $n^T m$ can define the type of the failure. For a pure mode-I , ***m*** is aligned with ***n*** and $n^T m = 1$,on the other hand for a pure mode-II, ***m*** is perpendicular to ***n*** then $n^T m = 0$. Alternatively the first condition is related to the tension failure and the second one indicates shear failure likewise between two amount is possible which shows mixed mode of failure in the element.

3. CONSTITUTIVE MODELS FOR REINFORCED CONCRETE

In this paper, for elasto-plastic calculation, the Drucker-Prager criterion which is suitable for such a concrete material has been used. In the case of plane stress, the constants of the Drucker-Prager criterion are so determined in this analysis that the failure surface of the Drucker-prager can go through the uniaxial tensile strength f_t and uniaxial compressive strength f'_c of concrete. In the case of compression in the failure surface, there will be a cut off to avoid more stress than f'_c (Fig. 2)

3.1 CONCRETE COMPRESSIVE RESPONSE

For compression response of concrete in the uniaxial test, linear form of relation between stress and strain has been chosen (Fig.3). In the compressive-compressive zone, two straight lines that go through f_c'', are adopted as the initial failure lines. f_c'' is the compressive strength when the tensile strain ε_{te} ,which is perpendicular to the compressive direction, is taken into account and can be evaluated by the equation $f_c''= \eta f_c'$ where:

$$\eta = \frac{1}{0.8 - 0.34(\varepsilon_{te} / \varepsilon_0)} \tag{11}$$

and ε_0 is the compressive strain at the peak stress f_c' and is usually taken as 0.002. The failure surface, when the concrete is untouched, is shown in Fig. 2.

3.2 CONCRETE TENSILE RESPONSE

A result of taking this approach is that the proposed concrete tensile response must now reflect the influence of the reinforcement. If the concrete is unreinforced, then the average tension in the concrete must reduce rapidly to zero. Fig.4 shows used tensile stress-strain model of concrete. The curve consist of two distinct branches. Before cracking the stress-strain relationship is essentially linear. Upon cracking ,however ,a drastic drop of strength occurs and the descending branch of the curve becomes concave. For simplicity, linear approximation of the descending branch of the curve has been chosen for the analysis.

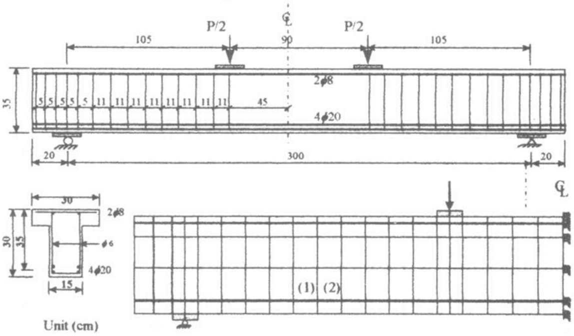

Fig. 5 Detailing of the specimen and finite element mesh

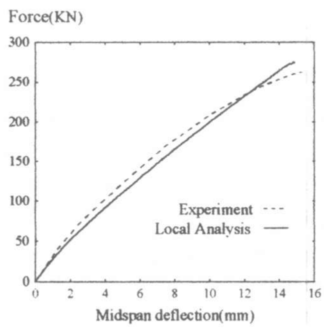

Fig. 6 Load versus midspan deflection

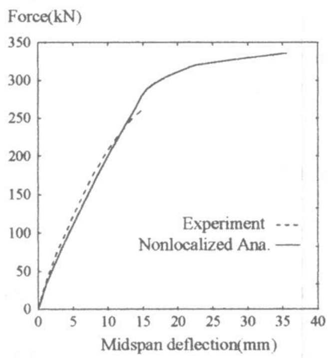

Fig. 7 Load versus midspan deflection

4. NUMERICAL INVESTIGATION

The constitutive models described and also mathematical expressions are implemented into the finite element formulation. All of the reinforced concrete beams are subjected to monotonic loading. Bond is assumed to be rigid in the nodal points and there is no slip between concrete elements and steel. The loads and the support reactions were applied as distributed forces to avoid stress concentrations.

4.1 ANALYSIS OF SIMPLE SUPPORTED BEAM

A T-beam with web reinforcement has been tested by Leonhardt and Walther[4] is analyzed here. The beam geometry and material properties are shown in Fig.5, along with the finite element mesh used to model the beam. In this model, tension reinforcement has been considered in one layer and all of the web reinforcing steel was treated as smeared and was included in the material properties of the reinforced concrete elements. The beam was heavily reinforced longitudinally in the tension zone and slightly reinforced by stirrups in the length of between loads and supports up to the end of the beam. Shear failure at the compression zone near the loading point has been reported for this beam.

The predicted responses for the midspan deflection versus total load in two cases are also shown, together with the observed response in Fig.6 and Fig.7. The agreement is very good for the calculation with the localization effects. As shown in figures, in the case of nonlocalized calculation, the beam is going to be failed in flexural mode and despite the big amount of steel, reinforcement is yielded. and it leads the beam to the flexural failure mode. In the other words, when we analyze the beams with stirrups, beams usually show more capacity against the loading than reality then the considering of localized form of failure would be more important in this kind of beams. In the localized calculation, beam is failed in shear and stress in the steel is less than yielded point. Shear failure happens in this beam while diagonal cracks between load and support developed upon compression zone near to the loading points.

For the sake of the limit of this paper, deformed shape and crack pattern were not presented but in the case of localized calculation, sever cracks occurred in element (1) and (2) which had much more better agreement with experiment's crack pattern at failure. For more detail, elements (1) and (2) have been considered. To define mode of failure for these two elements, localization vector $\boldsymbol{n}$ and vector $\boldsymbol{m}$ can be found by the Eq.8 which $\boldsymbol{n}^T.\boldsymbol{m}$ gives type of failure in the level of each element. For element (1) we have $m=\begin{Bmatrix}0.799\\-0.602\end{Bmatrix}$, $n=\begin{Bmatrix}0.944\\-0.329\end{Bmatrix}$

$n^T.m=0.952$ which gives the angle of $\boldsymbol{n}$ and $\boldsymbol{m}$ equal to 16.7 degree .It means this element fails in tension. For element(2) also we have:

$m=\begin{Bmatrix}0.839\\-0.545\end{Bmatrix}$, $n=\begin{Bmatrix}0.944\\-0.329\end{Bmatrix}$ $n^T.m=0.941$ which gives the angle of $\boldsymbol{n}$ and $\boldsymbol{m}$ equal to 13.8 degree then both element have failed in tension.

4.2 ANALYSIS OF DEEP BEAM

Paiva and Siess[5] tested a series of deep beams in 1965 to study the stress distribution and behavior of deep beams in shear. 19 beams were tested and the major variables involved in the study were the amount of tension reinforcement, the concrete strength, the amount of web reinforcement and the span-depth ratio. Beam G33S-31 is analyzed here.

The beam, shown in Fig. 8, was supported on a span of 61 cm(24.0"). The beam is reinforced in both tension and compression zones with two straight bars placed in single layers near the bottom and top and extending the full length of the beam. The tension reinforcement is anchorage by welding 2 inch by ½ inch steel plates at the end of the bars and make rigid bond for reinforcement. The experimental load-deflection curve, together with the prediction from analysis in two case of with and without localization effects are shown in Fig.9. The agreement with experiment result is very good for localized calculation.

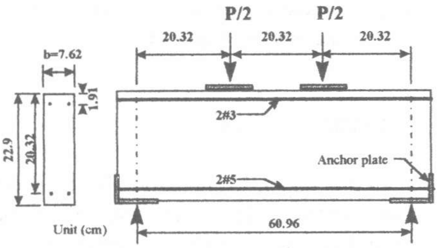

Fig. 8 Detailing of the specimen

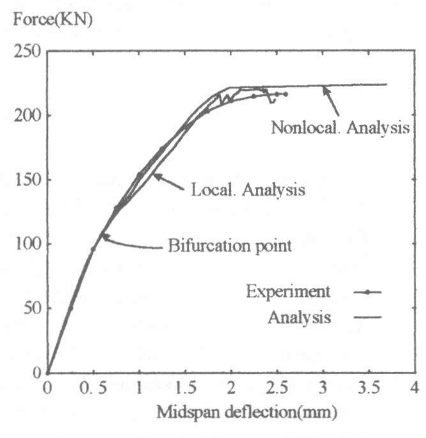

Fig. 9 Load versus midspan deflection

The beam was failed in shear as reported in the experiment result. The strain in steel goes up and before yielding will come back because of the concrete crash in the case of local analysis. As seen in the Fig. 9, after a few steps of calculation, in the bifurcation point, localization happens in some elements and beam will be softer against loading than standard finite element calculation. This phenomenon is due to shear band formation and cause more deflection in the same load. Fig.10 shows the deflected shape and crack patterns for both localized and nonlocalized calculation. The displacements and crack width are magnified by factors of 20 and 30 respectively. The cracks are oriented with the shear band direction which were determined by Eqs.5 and 6 and in fact, this figure shows the shear band patterns in the localized elements based on Eqs.5 and 6 and also strain distribution. For elements (1)and(2) we have $n^T.m = 0.612$ and $n^T.m = 0.549$ which show mixed mode of failure for both elements. The figure also shows the differences between crack width in two cases and it is clearly seen that the cracks width in the localized analysis are entirely severe than in other one which have very good agreement with experiment. The crack directions have also very good agreement with observed cracks in experiment.

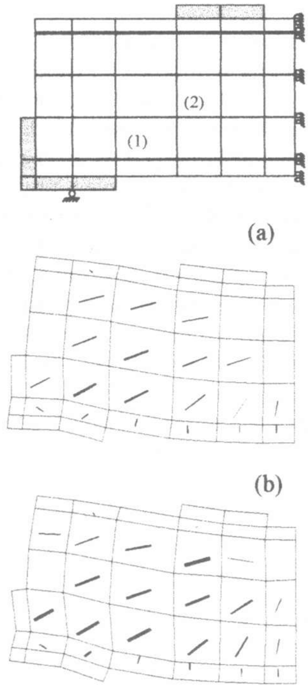

Fig.10 Finite element mesh and deflected shapes and crack patternsin two cases: (a) Nonlocalized analysis and (b) Localized analysis both at the point of (1).

5. CONCLUSION

In this paper, localization phenomenon in RC structures based on finite element method with embedded localization zones has been considered. The base of this study is to calculation of $\boldsymbol{B}$ matrix through dividing the element to two localized and nonlocalized zone. After bifurcation point, a jump in the strain field happens and the element will be divided to two localized and nonlocalized part. Localization occurs in an element when acoustic tensor ceases to be positive definite. With acoustic tensor, we can calculate the direction of cracks. At the end, numerical analysis of two beams showed the ability of the method to simulates shear failure in the RC elements.

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