論文 Solidification Model of Hardening Concrete Composite for Predicting Creep and Shrinkage of Concrete

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ABSTRACT: The prediction of both creep and shrinkage of hardening young concrete is attempted using a solidification model based on micro-physical information. The combined effect of external loads and pore water pressure caused by the surface tension is treated as the driving force for concrete deformation. The solid model deals with cement paste as the solidified finite fictitious clusters having each creep property. Aggregates are idealized as suspended continuum media of perfect elasticity. The combination of both phases may create the overall features of concrete composite. One important phenomenon is that Pickett's effect results as a natural outcome from the model computations.
KEYWORDS: young concrete, shrinkage, creep, multiphase material, microphysics, solidification, Pickett's effect

1. INTRODUCTION

Needs of predicting behaviors of early aged concrete have always been encountered, especially, in the case of massive concrete such as dams or raft foundation. These types of structures are always subjected to rather huge temperature gradients leading to thermal cracking. This makes it of great importance to be able to predict correctly the early age behavior of these structures. In the past, it was somehow difficult to treat both shrinkage and creep of young concrete based on the unified approach of material science. Now, more microphysical information is available like temperature, hydration ratio, porosity, pore water pressure, transient water content, isotherm and others. Based on this, the prediction of both creep and shrinkage of hardening young concrete is attempted. One advantage of this research is that it attempts to deal with these behaviors using only one microphysical model in a unified manner without any separation of creep and drying shrinkage. Then, the nonlinear coupling complexity named Pickett's effect [1] can be consistently simulated with the unified manner rooted in the thermo-hygro-physics. The point of importance is that this effect may be computed just as a natural behavior of cementitious materials with microporosity.

2. ANALYTICAL MODEL

The total deformation of concrete is divided into two components, that is to say, volumetric component and deviatoric component.

2.1 VOLUMETRIC DEFORMATION

Concrete is idealized as a two-phase solid dispersion system, namely cement paste and

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aggregate. Aggregate is assumed as linear elastic material suspended in the cement paste. The solidification concept [2,3] is used to represent the behavior of cement paste. The growth of cement paste is idealized by the formation of finite fictitious clusters or layers. When, the total volume of cement paste is V_{cp} and the effective volume solidified at time t is $V(t)$, the ratio of hydration controlling the solidification of layers is $\psi(t) = V(t) / V_{cp}$. New layers solidify one by one according to the hydration development. It is assumed that the new layers solidified join the existing ones in parallel coupling. Fig. 1 shows a schematic representation of the system at an arbitrary stage.

Concerning the coupling of aggregate and cement paste, virtual work principle (basically, Green's formula for conversion of volumetric divergence to the surface outer stream integral) was utilized for theoretically deriving the mean stress and strain on the aggregate and cement paste. According to this, the following two equations of phase requirements and two constitutive laws can be obtained.

$$\bar{\sigma}_o = \rho_{ag}\bar{\sigma}_{ag} + \rho_{cp}\bar{\sigma}_{cp} \qquad (1)$$

$$\bar{\varepsilon}_o = \rho_{ag}\bar{\varepsilon}_{ag} + \rho_{cp}\bar{\varepsilon}_{cp} \qquad (2)$$

$$\bar{\varepsilon}_{cp} = f(\bar{\sigma}_{cp}) \qquad (3)$$

$$\bar{\varepsilon}_{ag} = \frac{1}{3K_{ag}}\bar{\sigma}_{ag} \qquad (4)$$

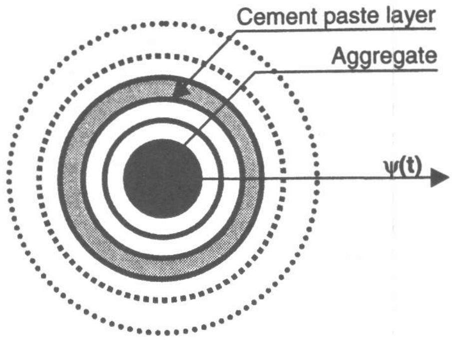

Fig. (1) Schematic representation of aggregate and cement paste

Where $\bar{\sigma}_o$, $\bar{\sigma}_{ag}$ and $\bar{\sigma}_{cp}$ are the average volumetric stresses on concrete, aggregate and cement paste, respectively, and $\bar{\varepsilon}_o$, $\bar{\varepsilon}_{ag}$ and $\bar{\varepsilon}_{cp}$ are the average volumetric strains, ρ_{ag} and ρ_{cp} are the volume fractions of aggregate and cement paste and K_{ag} is the volumetric stiffness of aggregate.

Capillary tension has been introduced by many of the past researchers as the source of shrinkage behavior [4]. Under this assumption the volume change and the deformation of cement paste will occur due to the surface tension force of capillary water across curved meniscus. In this study, the combined effect of external loads and pore water pressure created by the micro-scale surface tension is treated as the driving force for the deformation of concrete. Accordingly, the total average volumetric stress on concrete $\bar{\sigma}_{ot}$ is calculated by using eq. (5) as,

$$\bar{\sigma}_{ot} = \bar{\sigma}_o + P\,(\text{pore water radius, surface tension of liquid water}) \qquad (5)$$

where P denotes pore water pressure computed by the thermo-hygro-physical approach [5].

Here, there are 6 variables which are $\bar{\sigma}_{ot}$, $\bar{\sigma}_{ag}$, $\bar{\sigma}_{cp}$, $\bar{\varepsilon}_{ag}$, $\bar{\varepsilon}_{cp}$ and $\bar{\varepsilon}_o$. The first 5 are independent variables but there are only 4 equations. Thus, one more governing equation to specify the multi-phase system of interaction is required. As expressed by Maxwell and Kelvin chains of viscous continuum, there can be parallel system and serial one. If the cement paste matrix would be a perfect liquid losing resistance to the shear deformation, $\bar{\sigma}_o = \bar{\sigma}_{cp}$, where, shear stiffness of cement paste G_{cp} becomes null. This system corresponds to Maxwell chain of serial system of elements. On the contrary, if shear stiffness G_{cp} is infinitely large, it brings no shear deformation at all, and the deformed shape with volumetric expansion or contraction is perfectly similar to the referential shape at an initial time. Thus, aggregate and cement paste deform with perfect proportionality or, $\bar{\varepsilon}_o = \bar{\varepsilon}_{cp}$. This system corresponds to Kelvin chain of parallel system of elements. The actual case is somehow in between these two cases. According to this, the fifth equation of concrete composite isotropy is associated with paste shear rigidity and related degree of parallelism. One of possible formulas is Lagragian method of linear summation as,

$$(\frac{\bar{\sigma}_{ag} - \bar{\sigma}_{cp}}{G_{cp}}) + (\bar{\varepsilon}_{ag} - \bar{\varepsilon}_{cp}) = 0 \tag{6}$$

where, this equation satisfies the above stated extreme cases in parallel and serial figures of composite. By adding this one to the other 4 equations, the simultaneous equations get mathematical completeness. That is, number of variables is equal to the number of equations.

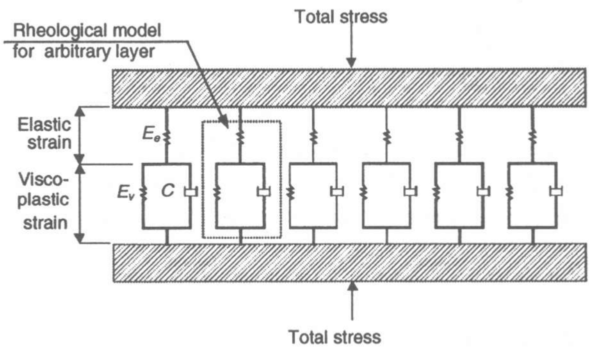

Fig. (2) Rhelogical model for cement paste layers

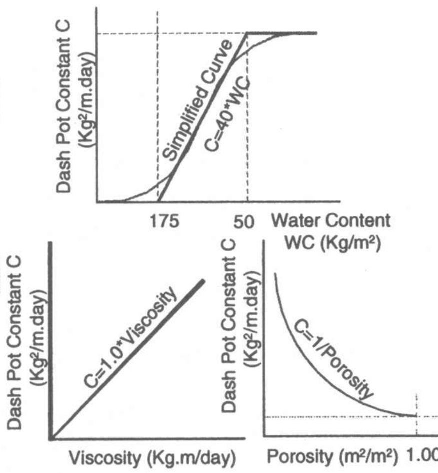

Fig. (3) Variation of dash pot constant against water content, viscosity and porosity

As shown in Fig. (2), the layers of cement paste are assumed to join in parallel where each layer is represented using two springs and a dashpot. Thus, the total average stress in cement paste at a certain time is the summation of stress in all individual layers found at that time and the average strain in cement paste is equal to strain induced in each layer. It should be noted here that when a new layer is just formed it should be stress free. Hence, the stress developed in each infinitesimal layer should be a function of the time when this layer gets solidified. Let S_{cp} denote the average stress in a general layer, t is the time and t' is the time when this layer is solidified. Then we have, $S_{cp} = S_{cp}(t,t')$ and $S(t,t) = 0$. The constitutive relation for each layer can be obtained using the rate type creep law as follows.

$$(1 + \frac{E_v}{E_e})S_{cp}(t) + \frac{C}{E_e}\frac{dS_{cp}(t)}{dt} = E_v \bar{\varepsilon}'_{cp}(t) + C\frac{d\bar{\varepsilon}'_{cp}(t)}{dt} \tag{7}$$

where, $\bar{\varepsilon}'_{cp}(t)$ is the strain in a general layer, E_e and E_v are the stiffness of elastic and plastic springs, respectively. C is the constant of the dashpot fluid and it is related to the water motion through pores associated with thermo-hygro-physical requirement. The average stress in the cement paste is calculated from,

$$\bar{\sigma}_{cp}(t) = \int_{t'=0}^{t} S_{cp}(t',t)d\psi(t') \tag{8}$$

The stiffness of each layer is calculated such that the summation of stiffness over all layers at a certain time is equal to that of cement paste at this time. The plastic stiffness of each layer, E_v, is taken as 1/3 of the total stiffness of the layer. The dashpot constant, C, is assumed a function in water content, viscosity of paste water and porosity as shown in Fig. (3).

2.2 DEVIATORIC DEFORMATION

The same procedure used for volumetric component is used. However, neglecting the shear strain of the aggregate component and assuming free rotation of suspended particles reduces the previously used system of simultaneous equations to,

$$\bar{\gamma}_o \cong \bar{\gamma}_{cp} \quad \text{and} \quad \bar{\tau}_o \cong \bar{\tau}_{cp} \tag{9}$$

$$\bar{\gamma}_o = f(\bar{\tau}_o) \text{ or } \bar{\gamma}_{cp} = f(\bar{\tau}_{cp}) \text{ or } \bar{\tau}_{cp}(t) = \int_{t'=0}^{t} \tau_{cp}(t',t)d\psi(t') \tag{10}$$

$$(1+\frac{G_v}{G_e})\tau_{cp}(t) + \frac{V}{G_e}\frac{d\tau_{cp}(t)}{dt} = G_v\overline{\gamma'}_{cp}(t) + V\frac{d\overline{\gamma'}_{cp}(t)}{dt} \tag{11}$$

where $\bar{\tau}_o$ and $\bar{\tau}_{cp}$ are the average deviatoric stresses on concrete and cement paste, respectively, $\bar{\gamma}_o$ and $\bar{\gamma}_{cp}$ are the average deviatoric strains, τ_{cp} and $\bar{\gamma}'_{cp}$ are the deviatoric stress and strain in a general layer, G_e *and* G_v are the stiffness of elastic and plastic springs, respectively and V is the constant of the dashpot fluid.

3. ANALYTICAL STUDY BY THE PROPOSED MODEL

An analytical study of creep and shrinkage of concrete was done using the proposed model. The microphysical properties needed are calculated using a thermo-dynamical program DuCOM [5]. The model is implemented in a finite element that is dynamically linked with DuCOM. For verification the analytical results were compared with some experimental results from the literature. A parametric study was also conducted to decide the values of the model parameters. The stiffness of cement paste is calculated based on the hydration level and water cement ratio.

4. VERIFICATIONS

4.1 PURE DRYING SHRINAKGE BEHAVIOR

The analytical results obtained are compared with the experimental ones conducted by Ishida [6] as shown in Fig. (4). The specimen was sealed for 2 days then subjected to drying under RH 45%. Also, comparison is made with Khan [7] in Fig. (5). The specimen was sealed for 0.53 days then subjected to drying under RH 50%. Both figures show reasonable agreement between the computed results and the experimental data.

4.2 DRYING CREEP BEHAVIOR

Comparison is made with Khan's data [7]. The experimental conditions are the same as in the drying shrinkage. The specimen is loaded at the age of 16 hours with as stress level of 0.185. The results are shown in Fig. (6), which shows good agreement between the computed results and the experimental data.

4.3 BASIC CREEP BEHAVIOR WITHOUT WATER LOSS

Comparison is made with Khan [7]. The experimental conditions are the same as before except that the specimen is sealed throughout the analysis. Fig. (7) shows the results that are in good agreement with the experimental data.

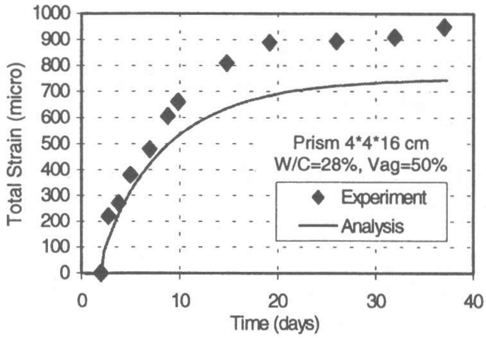

Fig. (4) Comparison with Ishida [6]
– Drying shrinkage

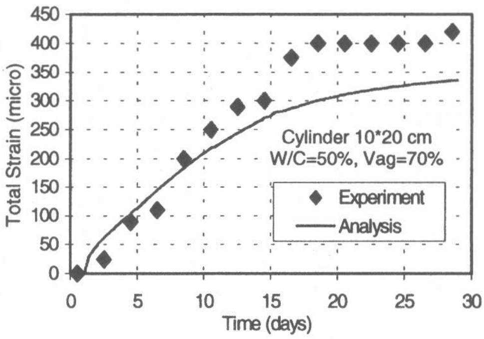

Fig. (5) Comparison with Khan [7]
– Drying shrinkage

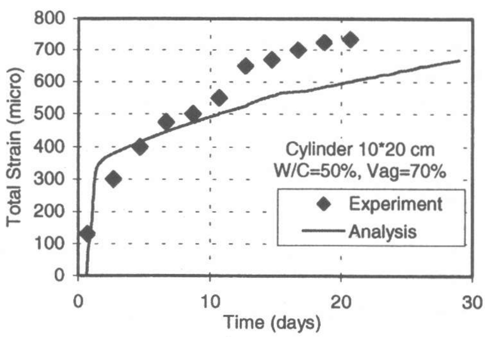

Fig. (6) Comparison with Khan [7]
– Drying Creep

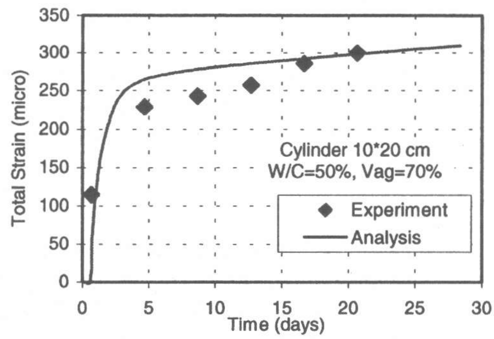

Fig. (7) Comparison with Khan [7]
– Basic Creep

5. PICKETT'S EFFECT

Many researchers have tried to explain the phenomenon of the Pickett's effect [8,9,10]. The Pickett's effect, also known as drying creep, is the excess of creep at drying over the sum of shrinkage and basic creep. Mostly, this effect is assumed due to ***Micro-cracking*** and ***Stress induced shrinkage***. Micro-cracking is caused by the non-uniform moisture distribution. When this moisture distribution approaches a uniform state, these micro-cracks cannot fully close. As a result the measured shrinkage of the specimen is always smaller than the true shrinkage of the specimen. Stress induced shrinkage is caused by the micro-diffusion transports of water between the capillary pores and the gel pores. This transport affects the deformation rate of the cement gel. The movement of water increases the breakage of the bonds, which are the source of creep. In this research, however, the Pickett's effect is shown to occur as a natural outcome from the concrete behavior. Fig. (8) shows a comparison between the results obtained from the model computation and the beam used by Pickett [1]. The figure shows fair agreement with the experiment. The dashed line shows the mathematical summation of deflection under pure drying shrinkage and under sealed with loading condition, that is, basic creep. The difference between this curve and the curve of drying and loaded specimen is very clear.

6. CONCLUSIONS

A solidification model based on microphysical information for the prediction of both creep and shrinkage of hardening young concrete information is presented in this study. The solid model deals with cement paste as the solidified clusters having each creep property. Aggregates are idealized as suspended continuum media of perfect elasticity. The combination of both phases is used to represent the overall composite under unstable transient situations at early age. The combined effect of external loads and pore water pressure is treated as the driving force for concrete deformation. Comparison with some data from the literature proved that the proposed model shows reasonable agreement with experiments. This shows that this framework can be a solid method of determination drying shrinkage, basic creep and drying creep. With some enhancement of the model parameters this model shows good future prospects. In addition, one important phenomenon is that Pickett's effect results as a natural outcome from the model computations.

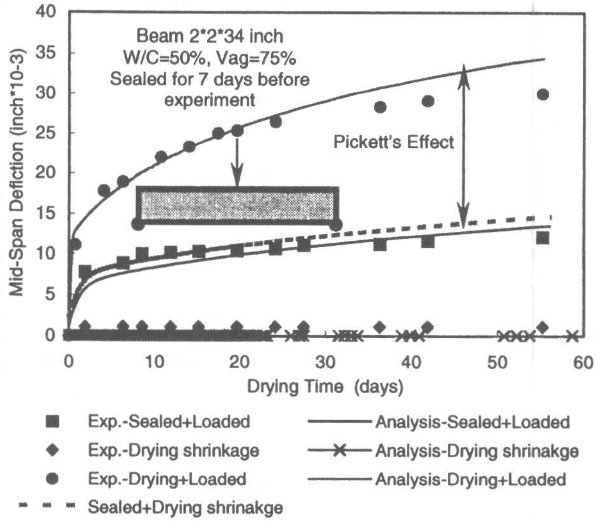

Fig. (8) Comparison with Pickett [1]

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