論文 Prediction of the Ultimate Flexural Strength of Externally Prestressed PC Beams

Thirugnanasuntharan ARAVINTHAN *1 Hiroshi MUTSUYOSHI *2
Atsushi FUJIOKA *1 Yoshihiro HISHIKI *3

ABSTRACT: The computation of the ultimate flexural strength of externally prestressed members requires a rigorous analysis methodology, since the tendon stress is member dependent rather than section dependent as in the beams with bonded tendons. As such, there is a necessity for a rational design equation to be used for the design of such beams. This paper describes the parametric evaluation conducted to investigate the influence of important factors that affect the ultimate strength of externally prestressed members. Based on these results a new design equation is proposed to predict the ultimate strength of prestressed beams with external prestressing.

KEYWORDS: external prestressing, flexural strength, prediction equation, prestressed concrete, tendon stress.

1. INTRODUCTION

Prestressed concrete bridges with external prestressing have become popular in the current construction trend due to its advantages such as a) reduced web thickness, b) possibility of repairing or re-strengthening of existing structures, etc. From the flexural analysis and design viewpoint, external tendons can be treated as unbonded tendons provided secondary effects and frictional forces at deviators are neglected. The ultimate flexural analysis of such beams offers an additional difficulty, in comparison to the beams with bonded tendons. The stress increase in the external or unbonded tendons beyond the effective prestress due to applied loading is member dependent rather than section dependent. In addition, it has been shown by Matupayont [1] and Alkhari [2], that the eccentricity variations could have a marked influence in the ultimate strength of externally prestressed beams. As such, it is necessary to consider the change in tendon position at ultimate state in the case of external prestressing for a better prediction of the ultimate flexural strength.

A nonlinear analytical methodology using a multi-level iterative technique has been developed to predict the complete flexural behavior of external tendons considering the above mentioned factors [3]. However, there is a need for a simplified design method to predict the ultimate flexural strength of such beams. Many prediction equations have been proposed by various investigators to compute the stress at ultimate in unbonded tendons. Nevertheless, it is a question whether these equations can be applied to external prestressing. To the best of authors' knowledge, the only equation that considers the eccentricity variation is the one proposed by Mutsuyoshi et.al [3]. Based on a similar approach, an attempt is made in this study to incorporate other factors that influence the ultimate tendon stress. An extensive parametric evaluation was conducted using the rigorous analysis and using these results, a modified equation is proposed to predict the tendon stress at ultimate in externally prestressed beams. The accuracy of the proposed equation is compared with the other equations.

*1 Graduate School of Engineering, Saitama University, Student Member of JCI
*2 Department of Civil Engineering, Saitama University, Member of JCI
*3 Kajima Corporation, Tokyo, Member of JCI

2. BASIS OF THE PROPOSED METHODOLOGY

The general form of the ultimate tendon stress for unbonded tendons can be expressed as follows:

$$f_{ps} = f_{pe} + \Delta f_{ps} \tag{1}$$

where f_{ps} is the ultimate tendon stress, f_{pe} is the effective initial prestress and is the Δf_{ps} increase of tendon stress.

In the existing prediction equations the estimation of Δf_{ps} varies based on the equations. An evaluation conducted by Naaman [4] show that there was room for improvement not only in terms of accuracy but particularly in accounting for the variables that were found to influence most the value of f_{ps}. As such, a new equation was proposed by Naaman [5] for the prediction of ultimate tendon stress in beams with unbonded tendons, based on the concept of strain reduction coefficient Ω_u. This reduces the member dependent analysis to a simplified section dependent analysis. The proposed equation can be expressed as follows, with the notations as defined in [5]:

$$f_{ps} = f_{pe} + E_{ps}\Omega_u\varepsilon_{cu}\left(\frac{d_{ps}}{c} - 1\right)\frac{L_1}{L_2} \leq 0.94 f_{py} \tag{2}$$

where Ω_u is the strain reduction coefficient defined as the ratio of the strain increase in unbonded tendon to the strain increase in concrete at the tendon level of the maximum moment section. Based on the data collected from 143 beam tests carried out by various investigators the value of Ω_u was proposed as follows:

$$\Omega_u = \frac{2.6\ [1.5]}{L/d_{ps}} \quad \text{for one-point loading}$$

$$\Omega_u = \frac{5.4\ [3.0]}{L/d_{ps}} \quad \text{for third-point or uniform loading} \tag{3}$$

This equation was later adopted by the AASHTO (1994) [6], with the recommended coefficients for Ω_u given within brackets in Eq. (3).

The applicability of the above equation to beams with external prestressing was carried out by Mutsuyoshi [3] and was found that it is necessary to take into account the change in tendon position at ultimate state. As a result, the concept of depth reduction factor R_d, that estimates the ultimate tendon position, was introduced for the prediction of the ultimate flexural strength of beams with external prestressing. Based on a parametric study, the following equation was proposed.

$$f_{ps} = f_{pe} + E_{ps}\Omega_u\varepsilon_{cu}\left(\frac{d_{pu}}{c} - 1\right) \leq f_{py} \tag{4}$$

where the ultimate tendon position d_{pu}, is given by the following expression:

$$d_{pu} = R_d\, d_{ps} \tag{5}$$

In the above equation, Ω_u and R_d were function of span-to-depth ratio (L/d_{ps}), deviator distance-to-span ratio (S_d/L) and loading span-to-span (M_d/L). The above equation has given the best results among all the prediction equations. However, the limitation is that it cannot be used for beams with combined prestressing consisting of internal bonded and external tendons. In the current study, the above approach is extended to incorporate the important parameters including the influence of internal bonded tendons.

3. PARAMETRIC EVALUATION

From the preceding discussion, it is evident that several factors influence the ultimate tendon stress of beams with unbonded or external tendons which in turn affects the ultimate flexural strength. An extensive parametric analysis was conducted considering the important factors. The variables and the number of cases considered in the parametric study are summarized in Table 1. A total of six parameters were selected covering the most practical range. Accordingly, the possible number of cases that need to be analyzed totaled 2592. However, using a sampling process, the actual number of cases analyzed were reduced to 864, one third of the above value. The material properties of reinforcements, prestressing steel and concrete were kept constant for all the cases. The layout of the PC beam model used in the analysis is shown in Fig. 1. It was a simply supported beam having a 'T' section. The external tendons were draped by two deviators and the loading was two point.

A nonlinear analytical program has been developed to predict the flexural behavior of PC beams with external prestressing. This methodology is based on a multi-level iterative technique taking into account the nonlinear constitutive models of materials, compatibility of deformation and change of eccentricity. The flow-chart describing this procedure is described in reference [3]. The predicted behavior by this program gave excellent correlation with experimental observations. As such, this program was used to investigate the influence of the above mentioned factors on the ultimate tendon stress in an externally prestressed member.

Table 1: Summary of variables used in parametric evaluation

No.	Description of variables	Range	Increment	No. of cases
1	Span-to-Depth ratio (L/d_{ps})	5 - 35	5	6
2	Loading span-to-Span ratio (L_p/L)	0.00 - 0.33	0.11	4
4	Bonded-to-Total tendon area ratio ($A_{ps,int}/A_{ps,tot}$)	0.00 - 0.75	0.25	4
3	Deviator distance-to-Span ratio (S_d/L)	0.33 - 0.67	0.17	3
5	Prestressing steel ratio (ρ_p) (in %)	0.25 - 0.45	0.10	3
6	Reinforcing steel ratio (ρ_s) (in %)	0.00 - 0.62	0.31	3
			Total number of combination	**2592**
			Actual number of cases analyzed	**864**

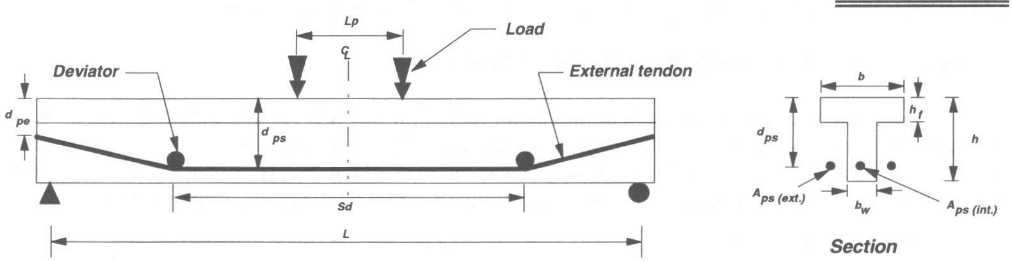
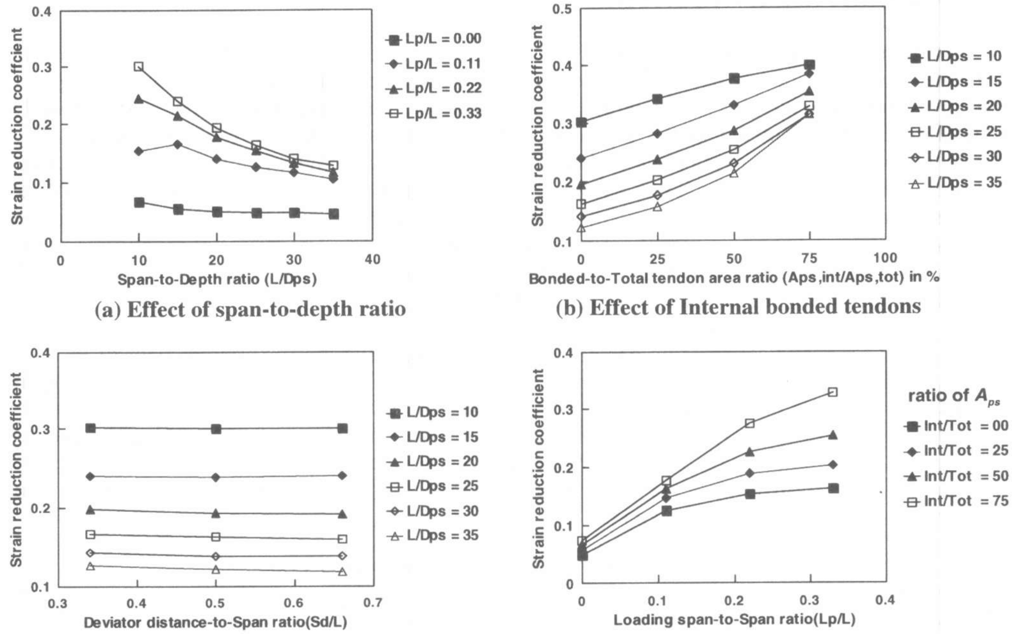


Fig. 1 Model of PC beam used in the parametric evaluation

4. INFLUENCE OF VARIOUS PARAMETERS

4.1 EFFECT ON STRAIN REDUCTION COEFFICIENT Ω_u

The influence of important parameters on the strain reduction coefficient Ω_u is illustrated in Fig. 2(a-d). From Fig. 2(a), it can be seen that the span-to-depth ratio (L/d_{ps}) greatly influences Ω_u. With increasing L/d_{ps}, the value of Ω_u reduces drastically in a hyperbolic manner. As such it can be said that Ω_u is inversely proportional to L/d_{ps}. As seen from Fig. 2(b), Ω_u increases with increasing ratio of internal bonded tendon ($A_{ps,int}/A_{ps,tot}$), in an almost linear manner. Judging from this it can be said that



(a) Effect of span-to-depth ratio **(b) Effect of Internal bonded tendons**

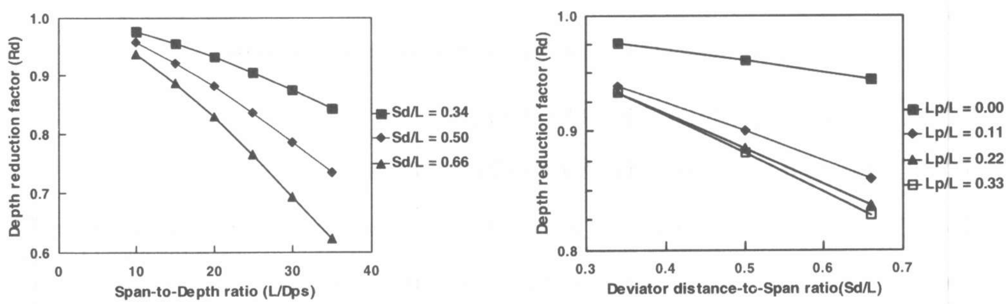
(c) Effect of deviator distance **(d) Effect of loading span**

Fig. 2 Influence of various parameters on strain reduction coefficient Ω_u

Ω_u is linearly proportional to $A_{ps,int}/A_{ps,tot}$. Fig. 2(c) shows that the value of Ω_u with deviator distance is nearly the same for different values of S_d/L. As such, it can be concluded that the influence of deviator distance on Ω_u is not significant. Similarly, the effect of ρ_p and ρ_s were negligible and are not presented here due to space limitations. As a result, these factors were neglected from the prediction equation of Ω_u. From Fig. 2(d) it can be seen that the variation of loading point on Ω_u is not linear. It is observed that the value of Ω_u for one point loading is considerably smaller than two point loading. This indicates that the effect of loading point should be considered as a separate issue.

4.2 EFFECT ON DEPTH REDUCTION FACTOR R_d

The influence of some parameters on the depth reduction factor R_d is illustrated in Fig. 3. It can be seen that the parameters L/d_{ps} and S_d/L are greatly affect R_d. In both cases, the value of R_d reduces with increasing ratio L/d_{ps} and S_d/L, in a nearly linear manner. It can be also noted from Fig. 3(b) that the reduction factor for one point loading is considerably different from the two point loading.



(a) Influence of span-to-depth ratio **(b) Influence of deviator distance**

Fig. 3 Influence of some parameters on depth reduction factor R_d

5. PROPOSED EQUATION

Using the results of obtained from the parametric evaluation, a multiple linear regression analysis was carried out considering various combinations of parameters. This analysis was done separately for the strain reduction coefficient Ω_u and the depth reduction factor R_d. Judging from Figs. 2 and 3, it was decided to separate the one point loading cases and two point loading cases. In addition, there were some cases where the external tendons yielded, which were excluded. The equations obtained for Ω_u and R_d through regression analysis are expressed as follows:

(a) Strain reduction coefficient Ω_u ;

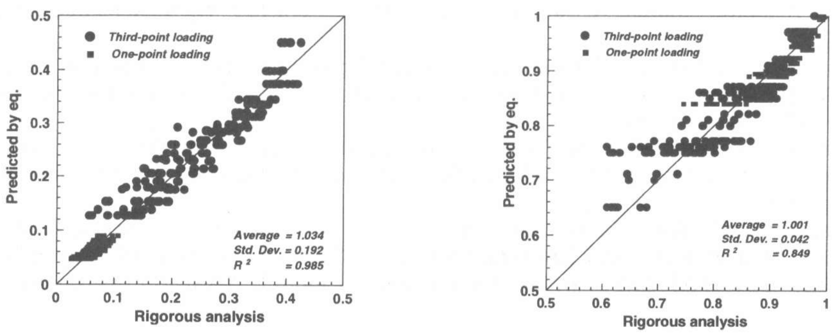
$$
\begin{aligned}
\Omega_u &= \frac{0.21}{(L/d_{ps})} + 0.04\left(\frac{A_{ps,int.}}{A_{ps,tot.}}\right) + 0.04 \text{ for one-point loading} \\
\Omega_u &= \frac{2.31}{(L/d_{ps})} + 0.21\left(\frac{A_{ps,int.}}{A_{ps,tot.}}\right) + 0.06 \text{ for third-point loading}
\end{aligned}
\qquad \textbf{(6)}
$$

(b) Depth reduction factor R_d ;

$$
\begin{aligned}
R_d &= 1.14 - 0.005\left(\frac{L}{d_{ps}}\right) - 0.19\left(\frac{S_d}{L}\right) \le 1.0 \text{ for one-point loading} \\
R_d &= 1.25 - 0.010\left(\frac{L}{d_{ps}}\right) - 0.38\left(\frac{S_d}{L}\right) \le 1.0 \text{ for two-point loading}
\end{aligned}
\qquad \textbf{(7)}
$$

By substituting Eqs. 6 and 7 in Eqs. 4 and 5, the expression for f_{ps} can be obtained. Considering the equilibrium of forces at the critical section, the neutral axis depth c can be computed and Eq. 4 will yield the value for f_{ps}. Once f_{ps} is known the ultimate flexural strength M_u can be calculated as explained in reference [5]. Fig. 4 illustrates the accuracy of the prediction of Ω_u and R_d using the proposed Eqs. 6 and 7 against the values obtained by rigorous analysis. From the statistical analysis, it can be seen that the proposed equation predicts Ω_u and R_d to a very good accuracy. It should be noted that Eq. 6 is expressed for one-point loading and third-point loading. In the case of uniform loading the equation for third-point loading can be used, since the moment diagrams of these two loading patterns are approximately the same. For two-point loading other than third point loading, the coefficients given in Eq. 6 for one-point and third-point loadings could be linearly interpolated to get the appropriate value of Ω_u.

The validity of the proposed equation was compared with the equation proposed by Naaman and the one adopted by AASHTO (1994). Fig. 5 shows the variation of predicted ultimate tendon stress



(a) Strain reduction coefficient Ω_u **(b) Depth reduction factor R_d**

Fig. 4 Comparison of the accuracy of the prediction

with the analytically obtained one for the range of parameters given in Table 1. It can be seen that the equation proposed in this study gave the best correlation compared to the others two equations. As such it is concluded that the proposed equation could predict the ultimate tendon stress of members with external and combined prestressing with a good accuracy. However, it is necessary to evaluate the proposed equation with the available experimental data to confirm its validity.

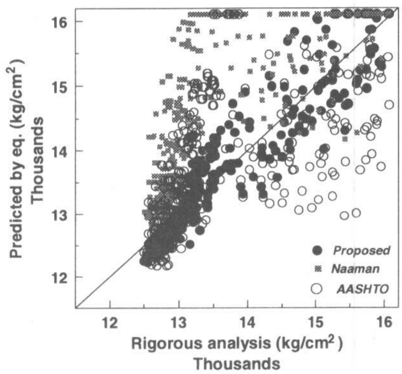


Fig. 5 Comparison of ultimate tendon stress

6. CONCLUSIONS

A parametric evaluation was conducted to study the influence of various factors that affect the ultimate tendon stress of beams with external prestressing. A new design equation was proposed for the prediction of the ultimate flexural strength of externally prestressed members. The conclusion from this study are as follows.

- The span-to-depth ratio was the most important factor that affect the ultimate tendon stress in the beams with external or unbonded tendons.
- The ultimate position of the external tendon is greatly influenced by the deviator distance-to-span ratio, thus affecting the ultimate flexural strength of such beams.
- The proposed equation can be used for member with external as well as combined prestressing. The prediction based on the new equation gave the best results compared to the other existing equations.
- It is proposed that further investigation should be carried out to study the applicability of the proposed design equation for continuous beams.

ACKNOWLEDGMENT

Sincere gratitude is expressed to Mr. Tadayoshi Niitsu, undergraduate student of Saitama University for his assistance in conducting part of the parametric study.

REFERENCES

1. Matupayont, S. and et.al., "Loss of Tendon's Eccentricity in External Prestressed Concrete Beam," Proceedings of JCI, Vol.16, No.2, June 1994, pp. 1033-1038.
2. Alkhari, F. M. and Naaman, A. E., "Analysis of Beams Prestressed with unbonded Internal or External Tendons," Journal of Structural Eng., ASCE, Vol.119, No.9, September 1993, pp. 2680-2700.
3. Mutsuyoshi, H. and et.al., "Flexural Behavior and Proposal of Design Equation for Flexural Strength of Externally Prestressed Concrete Members," Proceedings of JSCE, No.508/V-26, February 1995, pp. 103-111.
4. Naaman, A. E. and Alkhari, F. M. "Stress at Ultimate in Unbonded Post-Tensioned Tendons-Part 1: Evaluation of the State-of-the-Art," ACI Structural Journal, Vol.88, No.5, September-October 1991, pp. 641-651.
5. Naaman, A. E. and Alkhari, F. M. "Stress at Ultimate in Unbonded Post-Tensioned Tendons: Part 2-Proposed Methodology," ACI Structural Journal, Vol.88, No.6, Nov-Dec. 1991, pp. 683-692.
6. AASHTO, "AASHTO LRFD Bridge Design Specifications," AASHTO, Washington, D.C., 1994.