論文 Tension Stiffness Modeling for Cracked Reinforced Concrete Derived from Micro-Bond Characteristics

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ABSTRACT: The aim of this study is to get the spatial average stress- average strain relationships of both reinforcing bars and cracked concrete in RC members based on the local bond characteristics between concrete and re-bars. The computational basis is the local bond-slip-strain model [3]. In the computation the local stress and strain profiles of both re-bars and concrete between two adjacent cracks are computed. Using these profiles, the spatial average stress and average strains can be computed. The computation is also capable of predicting the ultimate average strain of re-bars. The comparison with the experiments shows good agreement.
KEY WORDS: Bond-slip-strain, tension stiffening, crack spacing.

1. INTRODUCTION

The tension stiffening effect represents the capacity of the concrete to carry the internal tensile force developing between adjacent cracks. The tensile force is solely carried by the steel reinforcing bars at the cracked section. This force is transferred to the concrete between adjacent cracks through the bond stresses between reinforcing bars and concrete. The tension stiffening effect is usually treated by assuming a relationship between the average concrete tensile stress and the average concrete tensile strain over a long-gauge length in the direction normal to cracks. At the same time, the stress-strain relationship of reinforcement has to be on average basis. As the stress in reinforcement embedded in concrete vary along re-bars, the average stress-average strain relationship of reinforcement is significantly different from pointwise one of bare bar. The rebar begins to yield at the position of concrete cracks prior to the remaining parts of re-bars. Therefore, the average yield stress will be lower than the yield stress of the bare bar [4]. After yielding some parts of reinforcement close to crack will be in the strain hardening zone, whereas the remaining parts which are far from the crack will still in the elastic zone. Therefore, the average response has a stiffness mixed from the elastic and the hardening one. Usually, a bilinear model is assumed for the average response of steel bars. The aim of this study is to get the average stress-average strain relationship of reinforcing bars as well as for concrete using a reliable physical bond model.

2. SPATIAL AVERAGED CONSTITUTIVE LAWS IN TENSION

2.1. BOND-SLIP-STRAIN MODEL:

Shima et al. [3] proposed a universal bond stress-axial slip-steel strain model for RC . The model

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offers unique relationship which expresses the bond characteristics derived from both pull out and axial tension tests. The constitutive law of bond is given by,

$$\tau(\varepsilon, s) = \tau_0(s)\ g(\varepsilon) \tag{1}$$

$\tau(\varepsilon, s)$: Bond stress, $\tau_0(s)$: Bond stress when strain is zero

$$\tau_o(s) = f'_c\ k\,[\ln(1 + 5s)]^c \tag{2}$$

$$g(\varepsilon) = \frac{1}{1 + 10^5 \varepsilon} \tag{3}$$

f'_c: Compressive strength of concrete, k: Constant=0.73, c: Constant=3
s: Non dimensional slip $=1000S/d$, S: Slip, d: Diameter of bar, ε : Strain of bar

2.2. BOND DETERIORATION MODEL:

Shima's model can not be applied to the bond deterioration zone where the "near crack surface effect" is predominant. In fact, the localization of plastic yielding is initiated from the bond deterioration zone. Thus, the modeling of bond close to cracks plays an important role for post-yield behaviour of RC in tension. Qureshi et al.[5] assumed in the RC joint model that the bond stress is linearly decreasing to zero at a distance 5 d from the crack surface, and that the bond stresses drops suddenly to zero at a distance 2.5 d from the crack surface due to splitting and crushing of concrete around the bar beside the crack surface. **Fig.1** shows a schematic drawing of bond deterioration model. The model is,

$$\tau(x) = \tau_{max} - \frac{\tau_{max}}{L_b}\{x - (\frac{L_c}{2} - L_b)\} \qquad (\frac{L_c}{2} - L_b \le x \le \frac{L_c}{2} - \frac{L_b}{2}) \tag{4}$$

$$\tau(x) = 0 \qquad (\frac{L_c}{2} - \frac{L_b}{2} \le x \le \frac{L_c}{2}) \tag{5}$$
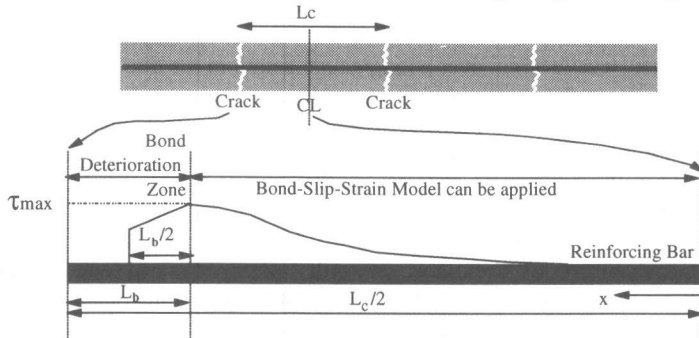
Fig.1   Bond Deterioration Model [5]

3. ANALYSIS

In order to get the steel stress profile, four equations should be solved simultaneously. By dividing the reinforcing bar between two adjacent cracks into small divisions or elements and studying the free body equilibrium of such elements, we get the following equilibrium equation,

$$\frac{d\sigma}{dx} = \frac{\pi d}{A_s}\ \bar{\tau} \tag{6}$$

where,

$\dfrac{d\sigma}{dx}$: Re-bar stress gradient along an axis, As: Re-bar cross sectional area, d : Diameter of reinforcing bar, $\overline{\tau}$: Average bond stress

The second equation is the bond-slip-strain model, together with the bond model in the bond-deterioration zone. The third equation is the slip compatibility equation. The slip is computed by integrating the strain over the length of the rebar starting from the midway between adjacent cracks, i.e. the slip at the midway between cracks is zero. Thus, we have,

$$S = \int \varepsilon \, dx \tag{7}$$

The fourth equation is the constitutive equation for the bare bar which represents the pointwise relationship between the re-bar stress and strain at each bar section,

$$\sigma = \sigma(\varepsilon) \tag{8}$$

Firstly, the crack spacing is equal to the total length of the specimen, and during analysis the local concrete tensile stresses are checked and a new crack is introduced whenever the stress reaches the cracking stress of concrete and a new average crack spacing is computed. Starting from the midway between two adjacent cracks, a finite segment with length Δx is studied. The boundary conditions are assumed by equating both the slip and the bond stress at the middle section to zero, and assuming a value to the strain at the middle. The four equations are simultaneously solved using an iterative procedure. Firstly, strain at the outer end of Δx is assumed. Then, the problem is to compute Δx satisfying the equations. By assuming Δx, the slip at the outer end is computed using equation (7), then the bond stress is computed using equation(1). The average bond stress is then computed, and Δx is computed using equation (6). If the difference between the assumed Δx and the computed one is within the proposed accuracy, the assumed value is accepted. If it is far from the proposed accuracy, Δx is reassumed again, till getting the correct value. Thus, finishing the computation of this division, the boundary conditions of the next division are defined and a similar computation procedure is followed. Hence, the strain and stress profiles of the steel reinforcement can be drawn. It results in the average stress and average strain as,

$$\overline{\sigma} = \frac{2}{L_c} \int_0^{\frac{L_c}{2}} \sigma(x) \, dx \cong \frac{2}{L_c} \sum_0^{\frac{L_c}{2}} \sigma(x) \Delta x \tag{9}$$
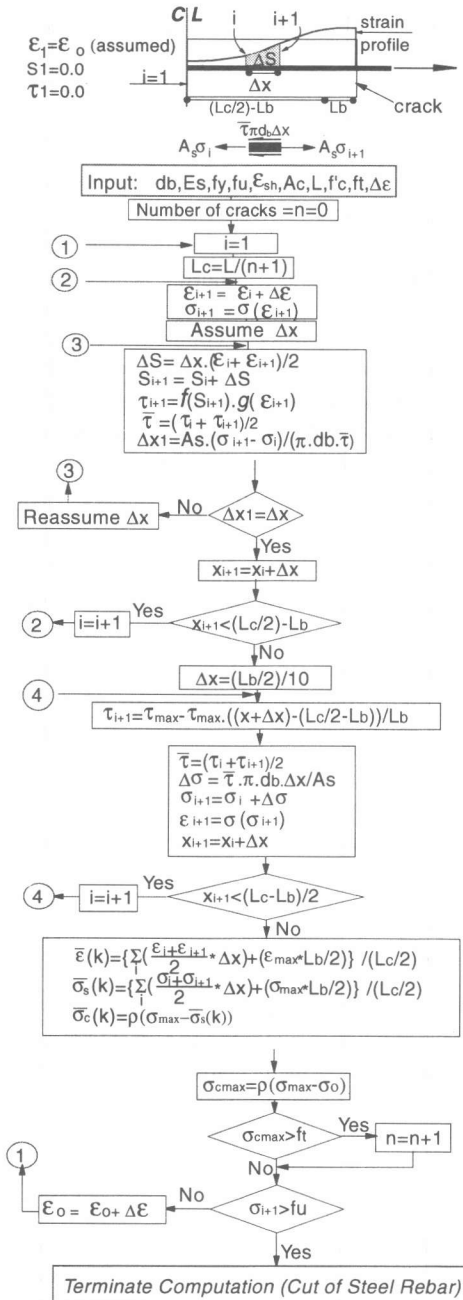
$$\overline{\varepsilon} = \frac{2}{L_c} \int_0^{\frac{L_c}{2}} \varepsilon(x) \, dx \cong \frac{2}{L_c} \sum_0^{\frac{L_c}{2}} \varepsilon(x) \Delta x \tag{10}$$

By computing the stress profile of reinforcement, the stress profile of concrete is obtained by subtracting the reinforcement force profile from the total force which equal to the re-bar force at the cracked section. Then, the average stress of concrete is also mathematically defined. A summary of the computation procedure is shown in **Fig.2**. A comparison with the experiments by Shima [3]is shown in **Fig.3** and **Fig.4**. The analysis agreed well with the reality .
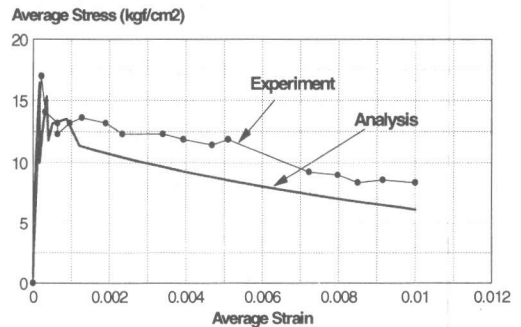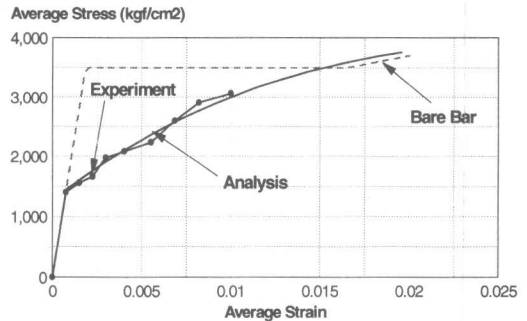
4. MEMBER ANALYSIS

As an application to the tension stiffening analysis, a FEM analysis was carried out to two cantilever columns tested under cyclic loading [7]. First, the average stress -average strain relationship of re-bars was computed. Then it was approximated to a bi-linear model composed of two lines, one is the elastic line up to the average yield point and the other is the strain

hardening part. The strain hardening part is non-linear in reality, but for simplification it can be approximated to a straight line. The bilinear model was applied in the 3D finite element program COM3 [6]. The columns were analysed in use of 3D frame elements. The internal loop of steel histeresis is described by Kato's model [1], and cyclic concrete model follows the proposal by Okamura et al.[4]. The sections of the analysed columns are shown in **Fig.5.** The ultimate capacities of the two columns were designed to be the same. The analysis of these columns are shown in **Fig.6** and **Fig.7** in a comparison with the experimental results. There was a good agreement between the experiments and the analysis in the envelope curve and critical displacement. The failure of the specimen with high strength reinforcement was due to the tensile rupture of re-bars, whereas the failure of the specimen with the low strength reinforcement was due to the crushing of concrete. The low reinforcement strength specimen showed higher ductility.
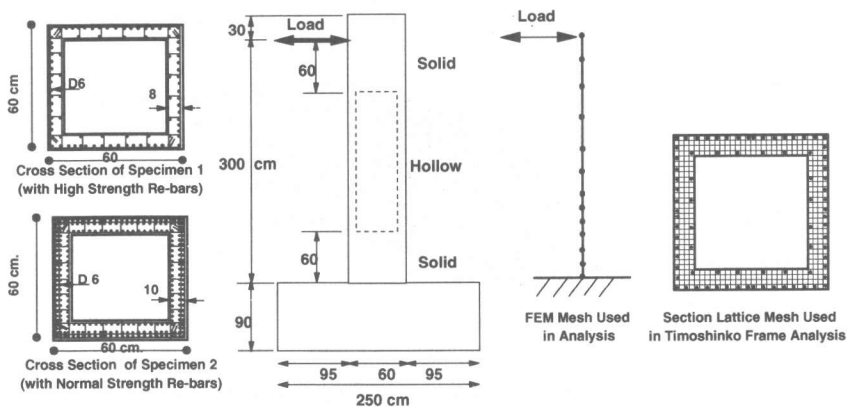

Fig.2 Flow Chart of Average Stress-Average Strain Computation

Fig.3 Reinforcement Response: A comparison with the experimental work of Shima et al [3]

Fig.4 Tension Stiffening: A comparison with the experimental work of Shima et al [3]

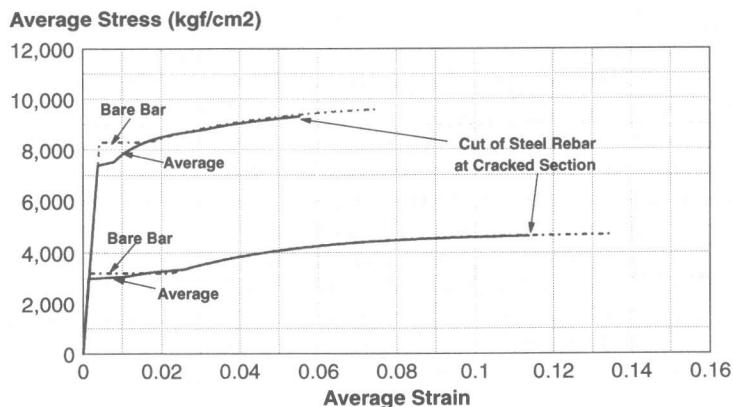**Fig.5** Dimensions and Reinforcement Details of Analysed Specimens[7].

Fig. 6 Computed Average Stress-Average Strain Relationship of Both High and Ordinary Strength Reinforcement
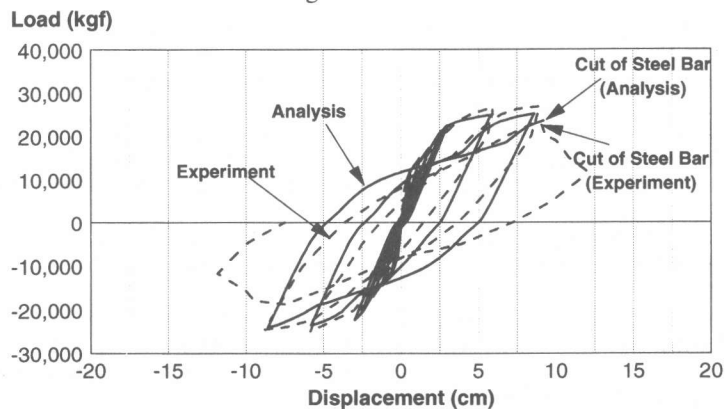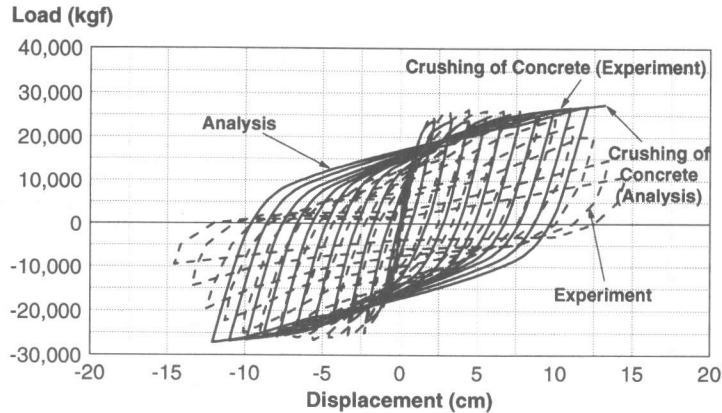

Fig.7.a Load-Displacement Relationship of Specimen with High Strength Reinforcement

Fig.7.b Load-Displacement Relationship of Specimen with Normal Strength Reinforcement (spalling of cover concrete is not considered)

5. CONCLUSIONS:

The conclusions of this paper can be summarised into the followings:

1. Based on the microscopic bond-slip-strain model and bond deterioration, the stress profile as well as the strain profile of reinforcing bars embedded in concrete can be computed. Hence, the macro average stress- average strain relationship of reinforcing bars as well as the tension stiffening of concrete can be computed. From the microscopic behaviour of reinforced concrete, the macroscopic behaviour can be detected .

2. The computations can predict the ultimate average strain of the reinforcement bars. The comparison with the experiments showed a good agreement with the measured ductility of specimens failed in tension. It has to be taken into account the possibility of such kinds of failure in which the reinforcement is ruptured. Herein a sample in which the reinforcement bars' ductility is not large was studied. This kind of failure exists also in the reinforced concrete columns with steel jacketing, in which the confinement of the concrete increases its capacity and ductility leading to cut of reinforcement re-bars prior to concrete crushing.

REFERENCES:

1. Kato, B." Mechanical Properties of Steel under Load Cycles Idealizing Seismic Action",CEB Bulletin D'Information, No.131, pp.7-27,1979.
2. Floegl, H. and Mang , H." Tension Stiffening Concept Based on Bond Slip" J. of Structural Div., ASCE Vol. 108, No. ST12, December, 1982.
3. Shima H., Chou L., and Okamura, H " Micro and Macro Models for Bond in Reinforced Concrete" Journal of The Faculty of Engineering ,The University of Tokyo(B), Vol.XXXIX, No 2, 1987.
4. Okamura, H. and Maekawa, K., "Nonlinear Analysis and Constitutive Models of Reinforced Concrete" ,Gihodo, Tokyo, 1990.
5. Qureshi, J. and Maekawa, K., "Computational Model for Steel Bar Embedded in Concrete under Combined Axial Pullout and Transverse Shear Displacement" , Proc. of JCI,Vol.15, No. 2, 1993, pp 1249-1254.
6. Maekawa, K., Irawan, P., and Okamura, H., "Three-Dimensional Constitutive Laws of Reinforced Concrete" International Conf. on Applied Concrete Mechancis APCOM, Seoul, 1996.
7. Masukawa, J., Amano, R., Suda, K., Ichinomiya, T.," Development of RC Column Members in Use of High Strength Reinforcement" Proc. of JCI, 1997