# 論文 A New Technique for Reinforced Concrete Beam Analysis Using the Modified Lattice Model

**Fawzy Mohamed El-Behairy*1 and Tada-aki TANABE*2**

**ABSTRACT.** A new technique to modify the lattice model is described by the authors. This new method significantly depends on the calculation of the minimum total potential energy of the structure starting from the elastic stage up to the failure stage inside each increment of the calculation. The adoption of the total potential energy, angle of inclination of the diagonals and the best position of the hinges along the depth of the beam are very important parameters affecting the results of the lattice model and are studied in this paper. The applicability of the modified lattice model is examined by proposed shear strength equations and existing experimental data.

**KEY WORDS:** Shear resisting mechanism, modified lattice model, arch element, total potential energy, subdiagonal element.

## 1. INTRODUCTION

It is generally agreed that the truss analogy concept is easily applied to the reinforced concrete structures. However, there exist several different truss models to analyze the shear resisting mechanism in the reinforced concrete beams. But in each model there are still some problems to be investigated. For example Lattice Model, which is first proposed by Niwa and et al [7] and extended later by the authors in 3-Dimensions [4][5], has several fundamental problems to be clarified. In his model arch member is a very important concept, because after yielding of shear reinforcement, the model could explain the increase in the shear capacity while the simple truss mechanism could not, specially in case of deep beam. Niwa [7] thus showed some important effect of the arch element in the shear carrying capacity.

In his work, Niwa determined the thickness of the arch element by minimizing the total potential energy for the whole structure. But, he did not give any physical explanation for the minimization of total potential energy and once the arch thickness is determined in the elastic stage, he kept it unchanged through out the whole loading history. Therefore, minimized potential thickness may be shifted during the loading stages. But its shifting is simply neglected.

Here, in this paper, we clarify this point in the first place and show the improved accuracy by performing minimization at every loading stages. Second problem is the rational reasoning for the strain incompatibility through the width of the beam by separating the arch member and the truss member within one beam. The third problem is the most appropriate discretization for truss member which is studied to find out the suitable form to apply our modified model. Also, the fundamental characteristics of the arch mechanism for shear resistance of reinforced concrete members is discussed, specially the strain values between the arch element and the diagonal element in the same cross-section of the beam are not in equal values[9]. The strain might not be uniform in the direction of member width[9].

Finally, in this paper, authors try to give rational reasoning or rational explanation for all these problems. Furthermore, basing on the modified and rational model we give some numerical calculation results which may be useful in its real application.

*1 Graduate Student, Department of Civil Engineering, Nagoya University, Member of JCI.

*2 Department of Civil Engineering, Nagoya University, Prof., Member of JCI.

## 2. RATIONAL REASONING OF THE SEPARATION OF THE ARCH ELEMENT AND TRUSS MEMBER WITH A BEAM

Structural action is normally a combination of series and parallel couplings of the cracking zones and the uncracked (elastic) zones. In the Modified Lattice Model, we simulated these zones with continuous pairs of tension and compression members. The arch member is considered as a very important element in our study, because it represents the core of the beam. A design code in Japan[1] assumes two dimensional stress field; but if a member section is wide enough, the stress might not be uniform in the direction of member width. It is also known experimentally by Ichinose [9] that the values of the strains or the stresses of the beam are not uniform along the width in the same cross-section. It means the stress-strain diagram is not constant in the direction of the width of the beam. In our model we separate the arch element and the diagonal element, each one of them has its stress-strain distribution. Arch element has the ability to resist a large portion of the applied load. So it is very important to look for the change of the thickness of the arch element during the calculation.

## 3. ADOPTION OF MINIMUM TOTAL POTENTIAL ENERGY

It has found that there is a relation between the thickness of the arch element and the corresponding total potential energy of the structure. Niwa et al [7] showed that if the ratio of the width of the arch element is assumed to be "t", the value of "t" is determined by minimizing the total potential energy for the whole structure. But in this work it is found that this thickness is increasing gradually during the loading from the elastic stage up to the complete failure of the beam. It means that the area of the diagonal members are decreased gradually during the loading from the elastic stage up to the complete failure. The physical explanation of the adoption of minimum total potential energy may be given first using a very simple spring model as shown its cross-section in Fig.1. The total potential energy for this model is as in equation (1). Substituting for $\sigma$ and $\varepsilon$ values we can get equation (2).

$$\pi = 1/2\int \sigma_1 \varepsilon dv_1 + 1/2\int \sigma_2 \varepsilon dv_2 - Pu \qquad (1)$$

$$\pi = -Au^2(1/2)[E_1 t/l + E_2(1-t)/l] \qquad (2)$$

From equation (2) $\pi$ is increasing monotonically with the increase of the area of the stiffer portion. Therefore, the stiffer portion should occupy the total area to make the potential minimum. However, our beam element is not exactly the same category. So, we show the real situation using the model shown in Fig.2. That is, common material exist as member (3) but each of member (1) and (2) consists of two elements as a truss element and arch element with a different rigidity and connected to member (3). The total potential energy of the structure is calculated from equation (3).

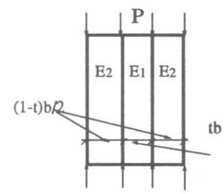


Fig.1 Cross- Section of a spring

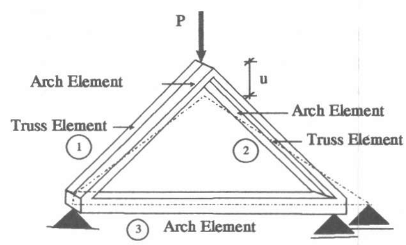


Fig.2 The Solved Example

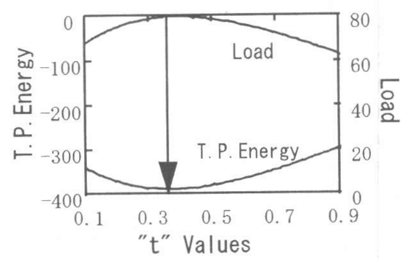


Fig.3 Behavior of the T. P. Energy and the Applied Load with "t"

$$\pi = 1/2\int \sigma \varepsilon dv - Pu \qquad (3)$$

where $u$ is the vertical displacement of the structure under the applied load "P". Taking $\partial\pi / \partial t = 0$ to get "t" value at the minimum total potential energy and substitute it in the energy equation. Fig.3 shows the relation of the total potential energy and the applied load "P" for the different values of "t". From this figure we can find that the point at the minimum total potential energy corresponds to the point of the maximum applied load at a definite value of "t". So, in the Modified Lattice Model we calculate the total potential energy for different values of "t" starting from 0.01 up to 0.9 with a very small increment. By minimizing these values of the total potential energy we can get the corresponding "t" value for each step of the calculation. Depending on this value of "t" we can calculate the area of the arch element and the subdiagonal elements for each step of calculation as in Fig.5. So, when we consider the value of the total potential energy and the corresponding area of the different elements inside each step of calculation we get the real response of the beam which becomes almost close to the experimental results along the different loading stages as we will see in the next section.

## 4. APPROPRIATE DISCRETIZATION METHOD FOR TRUSS MEMBER

Next thing to do is the clarification of the appropriate discretization of Lattice member which angle may be predetermined as 45 degrees. To investigate this problem three different truss models depending on the number of the pairs of diagonals along the depth of the beam are investigated. The three different forms are as shown in Fig.4. In Fig.4 the R.C. beam has been simulated under bending and shear as a simple truss components. The compressive stress in the upper part of the beam is resisted by concrete in the form of a horizontal strut with a cross-section area equal to the area of the upper rectangular in Fig.5. The tensile stress in the lower portion is taken by the bottom steel in the form of horizontal members, in addition to the horizontal concrete fibers in the lower part with a cross-section area equal to the area of the lower rectangular in Fig.5. To resist the shear forces inside the beam, the truss model has diagonal concrete tension and compression members with an area as shown in Fig.5 which be fixed after determined the value of "t" as mentioned before. Also the model has vertical steel members which represent the shear reinforcement in the web. Fig. 5 shows the schematic diagram of the cross section of a concrete beam modeled into the Lattice Model. The arch member is assumed to be a flat and slender one connecting the nodes at both ends with an area as shown in Fig.5. To study the applicability of the Modified Lattice Model we examined six different beams as shown in Table 1 using the above mentioned three different forms of truss models.

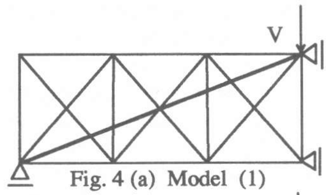


Fig. 4 (a) Model (1)

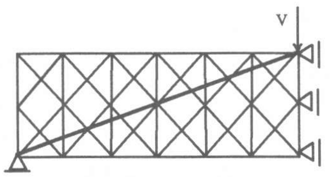


Fig. 4 (b) Model (2)

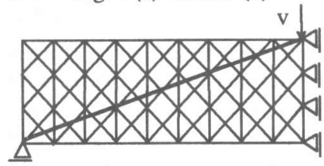


Fig. 4 (c) Model (3)

Fig. 4 Forms of Model of Simulation

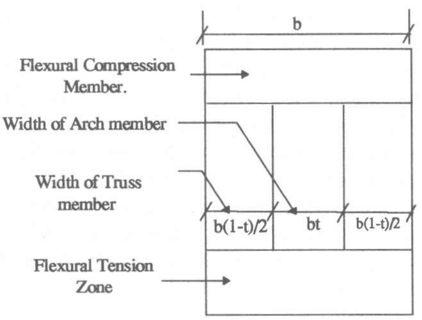


Fig. 5 Cross -Section of Concrete Beam in the Modified Lattice Model

**Table 1 The outline of Experimental Data**

| No | Cross Sec. | b cm | h cm | d cm | a/d | $f_c$ MPa | $A_s$ cm2 | $f_y$ MPa | $A_w$ cm2 | $f_{wy}$ MPa | s |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | R | 20.3 | 50.8 | 42.5 | 2.15 | 31.0 | 23.1 | 530 | 1.42 | 530 | 1.33 |
| 2 | R | 20.3 | 45.7 | 38.9 | 2.00 | 24.6 | 24.5 | 320 | 1.42 | 320 | 18.3 |
| 3 | R | 30.0 | 35.0 | 30.0 | 3.50 | 23.7 | 12.2 | 419 | 0.56 | 314 | 11.0 |
| 4 | T | 30.0 (15.0) | 35.0 | 30.0 | 3.50 | 23.7 | 12.2 | 419 | 0.56 | 314 | 11.0 |
| 5 | R | 45.0 | 60.0 | 52.5 | 2.86 | 43.9 | 95.7 | 383 | 1.43 | 355 | 25.0 |
| 6 | R | 45.0 | 60.0 | 52.5 | 2.86 | 66.2 | 95.7 | 383 | 1.43 | 355 | 15.0 |

Fig. 6 shows the comparison of the calculated results by the Modified Lattice Model and the normal Lattice Model with Clark's experiment[3] (No.4 in Table 1). Subsection diagonal members are increased from model (1) to model (2) to model (3), so the strain energy has been decreased for the decrease of the original length of failure elements. Also the cracking load decreased from model (1) to model (2) and model (3) for the decreasing of the elastic energy of the failure elements.

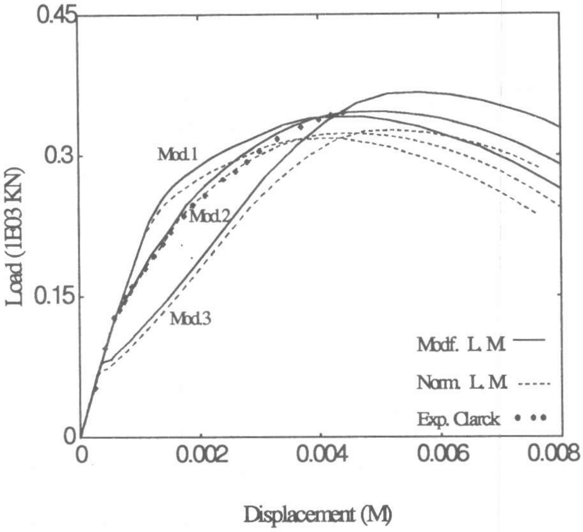


Fig. 6 Comparison With Experiment (N0. 4)

From the getting results we can analyze the behavior of the three forms of the model that under the applying load and after the cracking occur the neutral axis inside the longitudinal beam starts to move upward while the development of the cracks. The height of the development of the cracks depends on the cross-section of the beam and the value of the steel reinforcement inside the beam. In case of model (1) if a crack occurs, it means the depth of the crack equals the whole depth of the beam. After the initiation of this crack the complete failure takes place suddenly which is not logically, because experimentally, failure does not occurs suddenly. In case of model (2) If cracks happen it means the depth of the crack equals half of the depth of the beam only and the failure does not exist. This case looks logical and close to the experimental behavior of the reinforced concrete beams. Therefore, in case of model (3) the depth of the first crack equals 1/3 of the total depth of the beam. In this case the development of the cracks is not similar to the experimental behavior of the beam. That is why we find that the numerical results are very close to the experimental results in case of model (2). So, it is preferable to use model (2) to implement the Modified Lattice Model.

The change of the thickness of the arch element are drawn in Fig. 7 and Fig. 8 for the beams of No.2 and No. 5 in Table 1 respectively during the calculations. The thickenss of the arch element is increasing gradually from the elastic stage, in which it remains constant , up to the complete failure of the beam. After the yielding point, the depth of the arch element is decreased due to the initiation of the cracks. So, the thickness of the arch element has increased gradually trying to continue the effect of the arch element up to the failure point. As have been discussed above, most appropriate truss discretization is model (2). This suggest that the probable arch width is between 0.4b in the early loading stage and increase with the load up to 0.7b.

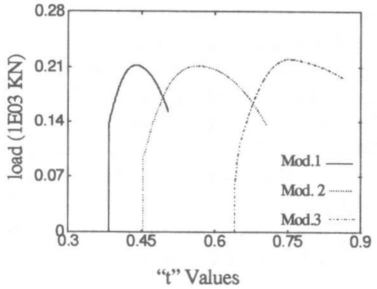


Fig. 7 Comparison of "t" Values for Three Models (No. 2)

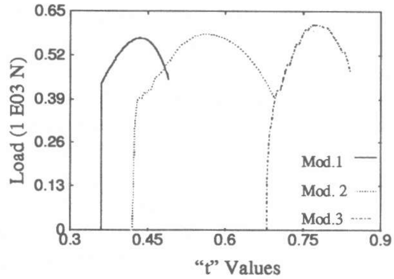


Fig. 8 Comparison of "t" Values for the three Models (No. 5)

## 5. APPLICATION OF THE MODIFIED LATTICE MODEL TO BEAMS WITHOUT WEB REINFORCEMENT

To examine the applicability of the Modified Lattice Model for concrete beams without web reinforcement numerical calculation is performed using the preferred model in Fig.4(b). The governing equation to calculate the shear strength of concrete beam without web reinforcement, is represented by Equation (4) which has been accepted by the Standard Specification of JSCE [8]. Here we will use Equation (4) to examine our Modified Model.

$$V_c(MPa) = 0.20 f_c^{1/3} P_w^{1/3} d^{-0.25}\left[0.75 + \frac{1.4}{a/d}\right] \quad (4)$$

where, $f_c$ is the compressive strength of concrete (MPa), $P_w$ is the reinforcement ratio (=100 $A_s/(b_w d)$, $d$ is the effective depth of a concrete beam (m), $a/d$ is the shear span-effective depth ratio. To examine the applicability of the suggested Modified Lattice Model for concrete beams without web reinforcement, the comparisons are carried out using equation (4) and also with the normal Lattice Model. Fig. 9 shows the

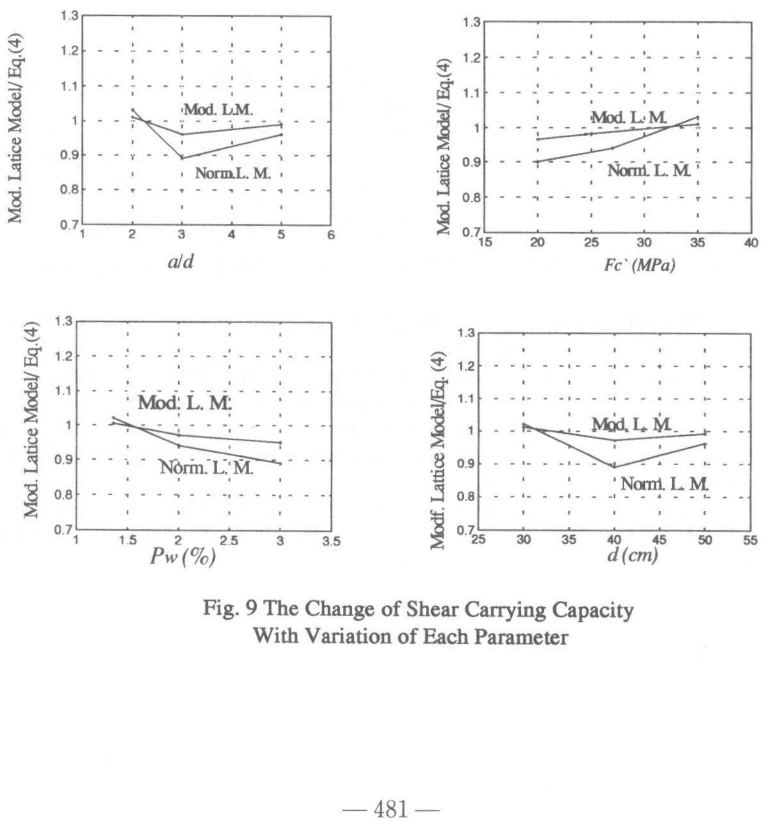


Fig. 9 The Change of Shear Carrying Capacity With Variation of Each Parameter

change of ratio of the results of predicted shear carrying capacity between the normal Lattice Model, Modified Lattice Model and equation (4) with the variation of different parameters. The parameters which are selected and combined were concrete strength, reinforcement ratio, effective depth and shear span-depth ratio. As seen from Fig.9, the predicted shear carrying capacity by the normal Lattice Model is smaller than that by equation (4), but the variation for the Modified Lattice Model is much smaller and admissible. Also from this figure the tendency of the prediction by the Modified Lattice Model is not necessarily similar to equation (4). The ratio is varied from 0.96 to 1.03, but in case of normal Lattice Model the ratio is almost from 0.88 to 1.1 [7]. Predicted shear failure mode by the Modified Lattice Model is the failure of the diagonal tension member which is corresponding experimental results. Consequently, it can be considered that the prediction of the shear carrying capacity by the modified lattice model is adequate.

## 6. CONCLUSIONS

In the newly developed Modified Lattice Model, a concrete beam subjected to shear force is converted into a simple truss and arch members by the consideration of the minimum total potential energy for the structure at each step of calculation. A non-linear incremental analysis is performed. Conclusions obtained from this research are as follows:

1. By minimizing the total potential energy of the reinforced concrete beam we get only one value for the thickness of the arch element, at which we get the stiffest case for the structure which is quite similar to the original response of the experimental analysis

2. The thickness of the arch member which plays a very important role in the Modified Lattice Model is increases gradually with the increase of displacement of the loading point after the initiation of diagonal cracks up to the complete failure of the beam.

3. The applicability of the modified lattice model is examined for beams with web reinforcement. It gives a quite close results when compared with the experimental results. Also, in case of beams without web reinforcement for deferent parametric conditions, the tendency of the prediction of strength by the Modified Lattice Model is very close to the equation of JSCE.

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