論文 Time-Dependent Deformation Analysis of RC Member Considering Non-Linearity of Concrete Creep

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ABSTRACT: This paper focuses on the applicability of the non-linear concrete creep law in the time-dependent deformation analysis of reinforced concrete members. To calculate the creep strain of flexural member using the creep law established on the axially loaded specimen, a parameter is introduced to account the steel restraining effect. In numerical calculation the method of time-discretization is used. The non-linear creep response is calculated due to the stress acting in an individual time-step and successively added. The present method gives better agreement with the experimental results in compare to the conventional methods of prediction.
KEYWORDS: concrete non-linear creep law, long-term deformation, method of time-discretization, RC member, shrinkage strain, steel restraining effect, stress redistribution

1. INTRODUCTION

The analysis of time-dependent deformation and its effect on the behavior of RC structural elements is necessary not only to solve the problems against failure but also to check the problems related durability, serviceability and long-term reliability. The problems encountered by the engineers is difficult not only because of the composite action of reinforcing steel and concrete, but also because of the non-elastic behavior of concrete itself [1]. The continuous redistribution of stress makes the assessment further complex.

The conventional creep deformation analysis of reinforced concrete members is based on linear relationship between creep strain and acting stress. This simplifies the complexity of analysis, but it has found that the experimental findings cannot be explained under the linear relationship. Attempts to quantify the time-dependent deformations have been made by some researchers [2],[3]. The calculation methods are based on the concept of creep coefficient and could not clarify the actual behavior of the RC member under long-term loading.

This paper summarizes the applicability of concrete non-linear creep law to assess the time-effects that could give a better representation of actual behavior in compare to the conventional creep analysis. For numerical prediction, a time-discretization method is used that considers state-of-stress and the elastic as well as creep response due to redistribution of stresses.

2. TIME EFFECTS AT RC BEAM SECTION

The stress and strain distributions through the depth of a cracked section of reinforced concrete flexural member have been shown in Fig. 1. The time-effect is to increase the strain of outermost compressed fiber from ε_i to $(\varepsilon_i+\varepsilon_t)$ and the tensile strain is assumed to remain unaffected by creep. In consequence, the neutral axis moves downward and its distance from

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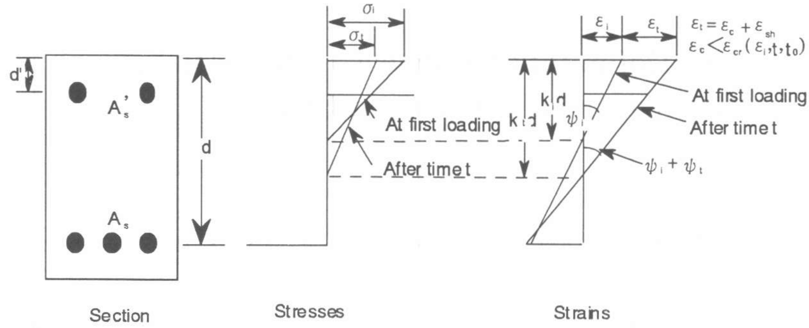

Fig. 1. Time effects at a cracked section.

the upper edge is equal to $k_t d$. This cause a new stress distribution in the compressed part of concrete, the stress will decrease from σ_i to σ_t.
The depth ratio of initial neutral axis is given as

$$k_i = \left[(\rho+\rho')^2 + 2(\rho+\rho'\frac{d'}{d})n\right]^{1/2} - (\rho+\rho')n \tag{1}$$

The depth ratio of neutral axis after the time t shown by Pieter C. Pretorius [3] is

$$k_t = \frac{k_i(1 + C_t)}{(1 + k_i C_t)} \tag{2}$$

In which, C_t is the long-term strain factor ($= \varepsilon_t / \varepsilon_{el}$) and $\varepsilon_t = \varepsilon_{cr} + \varepsilon_{sh}$.
The subscript i refers to immediate conditions and t to the conditions at the time t.

2.1 LONG-TERM DEFLECTION

If the curvature φ is known at all points x along the member, the deflection equation at any point of the member can be expressed as

$$\Delta = \int_L \varphi_x \, m(x) \, dx \tag{3}$$

where, m(x) is the bending moment at x due to unit load applied at the given point.

If the reinforcement is constant along the span and considering that the strain of tension reinforcement remains fairly unaffected by creep, the time-dependent deflection at time t from Eq. (3) after mathematical manipulation can be shown as

$$\Delta_t = \frac{C_t(1 + k_i C_t)}{1 + C_t} \times \Delta_i \tag{4}$$

Eq. (4) gives a simple multiplier to calculate the long-term deflection from the initial deflection provided the long-term strain factor of the top fiber C_t is known.

3. ALGORITHM FOR NUMERICAL PREDICTION

Strain is predicted by discretizing the load duration, the time period(t, t_0) is subdivided into a number of time-steps within which the stress is considered constant. The actual stress history is approximated by piecewise constant stress history, admitting stress changes only at discrete time points, i=1,2,...........n. The model stress history at a section of cracked member regarding to the time discretization is shown in Fig. 2.

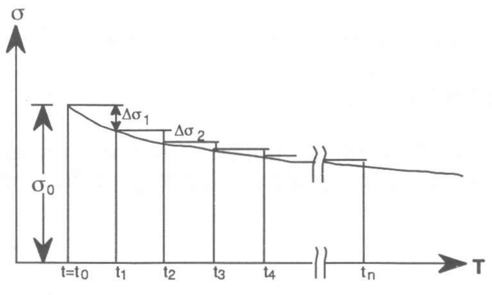

Fig. 2. Stress history by the method of time-discretization

3.1 ESTABLISHING THE STRESS HISTORY

As creep progress, the neutral axis in Fig. 1 migrates downward and compressive stress in the outermost compression fiber subsequently decreases. Since the external moment remains unchanged because of constant sustained load, with mathematical manipulation the change in stress within the time period(t_i, t_0) can be shown that

$$\Delta\sigma(t_i, t_0) = \frac{(k_t - k_i)(3 - k_i - k_t)}{k_t(3 - k_t)} \times \sigma_0 \qquad (5)$$

where, k_i refers to t_0 and k_t to t_i.

Stress decrement at the i-th step is $\Delta\sigma_i = \Delta\sigma(t_i, t_0) - \Delta\sigma(t_{i-1}, t_0)$ and stress sustains throughout the i-th step is $\sigma_i = \sigma_0 - \Delta\sigma(t_{i-1}, t_0)$.

3.2 NON-LINEAR CREEP ANALYSIS (NLA)

Creep strain due to the sustained stress of a certain time-step is independently calculated for the respective period. Accordingly, creep strain of time-step i belonging the period (t_i, t_{i-1}) is calculated considering the stress σ_i being applied at time t_0 sustains up to the time t_i and simultaneously a negative stress of equal magnitude prevails the period (t_{i-1}, t_0). Hence, creep strain at i-th step is $\varepsilon_{cr}(i) = \varepsilon_{cr}(\varepsilon_i, t_i, t_0) - \varepsilon_{cr}(\varepsilon_i, t_{i-1}, t_0)$. Where, ε_i is the elastic strain corresponding to the stress σ_i. Elastic strains as well as creep recovery due to the stress step down shown in Fig. 2 are superposed on the creep strain $\varepsilon_{cr}(i)$ to get the net creep strain for the i-th period. The strains are successively added to calculate the creep strain within the period (t, t_0). Hence, the history integral of creep strain is

$$\varepsilon_{cr}^{net}(t, t_0) = \sum_{i=1}^{n} \varepsilon_{cr}^{net}(i) = \varepsilon_{cr}^{net}(1) + \varepsilon_{cr}^{net}(2) + \cdots\cdots + \varepsilon_{cr}^{net}(n-1) + \varepsilon_{cr}^{net}(n) \qquad (6)$$

(1) Concrete non-linear creep law

The non-linear material creep law proposed by Sakata and Ayano [4] is modified to calculate creep strain of the reinforced concrete flexural member. The modification incorporates a parameter α representing the steel restraining effect and a factor to consider the effect of compression steel on the creep of concrete . Accordingly, creep of concrete under flexure is

$$\varepsilon_{cr}^{flexure}(\varepsilon_{el},t,t') = \varepsilon_{cr}^{prism}(\varepsilon_{el}/\alpha,t,t') \times \left(1 - \frac{\rho'}{2\rho}\right) \quad (7)$$

where, $\varepsilon_{cr}^{flexure}$ and ε_{cr}^{prism} are the creep strain under flexural and uniaxial stress, respectively.

3.3 CONVENTIONAL LINEAR CREEP ANALYSIS (CLA)

Conventional linear creep analysis implies validity of the principle of superposition. According to the Bolzmann's principle of superposition, creep strain regarding to the stress history shown in Fig. 2 is

$$\varepsilon_{cr}(t,t_0) = \frac{1}{E}\left[\sigma_0 \times \phi(t,t_0) - \sum_{i=1}^{n} \Delta\sigma_i(t_i)\{1 + \phi(t,t_i)\}\right] \quad (8)$$

(1) Concrete creep coefficient

In linear creep analysis the creep strain is assumed to be proportional to the acting stress. Considering the time of the application of load and the loading period the creep coefficient is

$$\phi(t,t_0) = 4.8(0.002t_0 + 1)^{-2.9}\left\{\frac{t - t_0}{1 + .01(t - t_0)}\right\}^{0.735} \quad (9)$$

3.4 SHRINKAGE STRAIN OF RC MEMBER

In a reinforced concrete member even with no external load, free shrinkage is restrained by the presence of reinforcement. Considering the shrinkage induced stresses in steel and the resulting creep of concrete, shrinkage of concrete of flexural member is

$$\varepsilon_{sh}^{flex}(t) = \varepsilon_{sh}^{plain}(t) + n^*\frac{\Delta\varepsilon_s}{I}(A_s y_1 - A_s' y_2)y \quad (10)$$

where, $n^* = n_0(1 + \phi_c)$, and $\Delta\varepsilon_s$ is the shrinkage induced steel strain.
Empirical concrete shrinkage law proposed by Sakata *et al* [5] used in the analysis is

$$\varepsilon_{sh}(t,t_0) = \varepsilon_{sh}^{\infty} \cdot \left[1 - \exp\{-.0186(t - t_0)^{0.5\phi}\}\right] \quad (11)$$

where, ϕ is a function of member V/S and ambient humidity, and t_0 is the start of drying.

4. EXPERIMENTAL PROGRAM

Eleven reinforced concrete beam specimens were subjected to sustained load to investigate the long-term deformations. The specimens were grouped into three according to the variation in tensile steel (ρ_s=.02, .04, .06), compression steel (ρ_s' =0, .006, .012, .017, .023) and the dimension of cross-section (d=6.2, 10.2, 14.2 cm). The constant sustained load was the 40%

of static failure load. Compressive strength of concrete at 28 days after casting was 35 MPa. Experiment was done in constant temperature (20±1°C) and relative humidity (68±5%) room. Observations were made for 100 days and measurements were carried out at 1,3,5,7 and 10 days of loading and every 5th day, thereafter.

5. COMPARISON OF PREDICTED RESULTS

The calculated data from the analysis are compared with the experimental data of all the beam specimens under controlled laboratory conditions. The experimental and calculated strain distribution at the mid-section of a doubly reinforced specimen is shown in Fig. 3. It is seen that the calculated distribution considering the non-linearity of creep agrees remarkably well with experimental findings. Conventional linear distribution differs significantly from the experimental results. The calculated long-term strain factors by the present non-linear analysis are correct within 20% percent of the measured values as shown in Fig. 4.

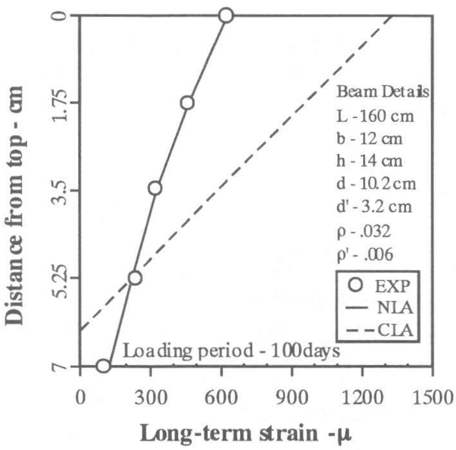

Fig. 3. Long-term strain distribution

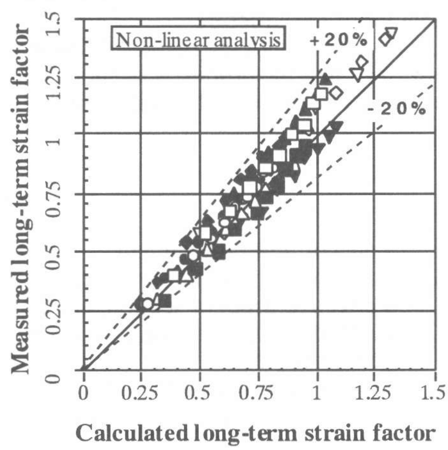

Fig. 4. Long-term strain factor by NLA

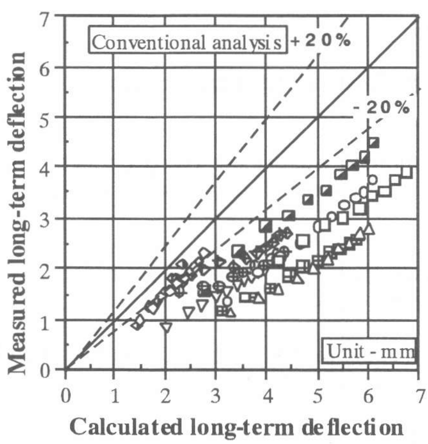

Fig. 5. Long-term deflection by CLA

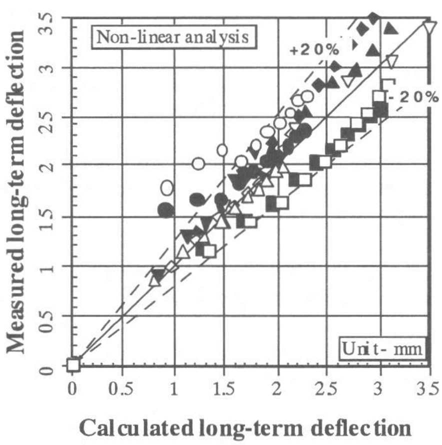

Fig. 6. Long-term deflection by NLA

In Fig. 5, calculated deflections by the conventional analysis is found significantly different from the measured deflections. Conventional analysis does not consider the steel restraining effect and the non-linearity of creep. Furthermore, for simplification the elastic and creep recovery due to stress redistribution is neglected. Calculated long-term deflections by the present analysis agrees very well with the experimental deflections as seen in Fig. 6.

6. CONCLUSIONS

Long-term deformation analysis considering the non-linearity of material behavior is proposed. In the analysis, effect of reinforcing steel to restrain the creep is considered. Redistribution of stress and the resulting elastic strain as well as creep recovery is considered to predict the net creep strain. Net strain of the individual time-steps are simply added to find out the total long-term strain. In contrast, the conventional creep deformation analysis is based on the material creep coefficient and the effect of tension steel to restrain the creep is not considered. The concept of creep coefficient is independent of acting stress hence, it cannot justify the actual behavior of the structure.

Comparison of the predicted results with the experimental results of various specimens has confirmed that the proposed method of analysis considering the creep non-linearity of concrete can be reliably used in the long-term deformation analysis at the design step.

NOTATION

A_s = area of tension steel, cm^2
A_s' = area of compression steel, cm^2
d = effective depth of section, cm
d' = effective depth to comp. steel, cm
n = modular ratio
y = distance from centroidal axis, cm
y_1 = distance of tension steel from centroidal axis, cm
y_2 = distance of compression steel from centroidal axis, cm

ε_{cr} = creep strain, μ
ε_{el} = elastic strain, μ
ε_{sh} = shrinkage strain, μ
ε_t = total long-term strain, μ
ρ = tensile steel ratio
ρ' = compression steel ratio
ϕ_c = creep coefficient

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