論文 Effects of the Position of Inflexure Points on the Ultimate Shear Capacity of Reinforced Concrete Beams with Vertical Shear Reinforcements

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ABSTRACT: This study presents the shear test on the reinforced concrete beam with vertical shear reinforcements, in which the position of inflexure points is treated as a main parameter. It is found that the inflexure points affect an ultimate shear capacity of the beam as much as 10% increment. Finally, the proposed formula for the ultimate shear capacity is established based on the experimental results.
KEYWORDS: shear, inflexure point, beam, estimation of ultimate shear capacity, vertical shear reinforcements.

1. INTRODUCTION

In most of current design codes for the reinforced concrete (RC) beam such as ACI code, JSCE code, the shear design formula for a beam with shear reinforcement is based on the test of the simple beam subjected to the two-point loading without the inflexure point within the shear span. It was reported in the previous study [1], [2] that for the RC beam without shear reinforcements, the presence of the inflexure point in the shear span increases the shear capacity due to the difference of the shear resisting mechanisms between beams with and without the inflexure points. In this paper, the influence of the position of inflexure points on the ultimate shear capacity of the RC beam with shear vertical reinforcements is investigated through the experiments on 10 RC beams with varied positions of inflexure points. By extending the formula proposed by Y. Aoyagi, et. al. [2], the formula to estimate the shear capacity of the RC beam with vertical shear reinforcements which can account for the effects of inflexure points is proposed based on the present experimental results.

2. OUTLINE OF THE EXPERIMENTAL METHOD

2.1 Beam specimens

A total of 10 beam specimens were tested in this study. Their configurations were the

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rectangular cross section of the constant size 15 cm (b) × 25 cm (h) and the length of 340 cm (Fig. 1). The compressive strength of concrete at the time of the test ranges from 290 to 347 kgf/cm². A longitudinal reinforcement had ratio of 4.18% and a yielding strength of 4800 kgf/cm². The diameter of 6 mm of vertical shear reinforcements (stirrups) was adopted and the yielding strength was 2900 kgf/cm² (type-I) and 3300 kgf/cm² (type-II). All of the beam specimens in Fig. 1 and Table 1 were so designed that the shear failure due to the yielding of stirrups occured prior to the flexural failure. It is noted that as shown in Fig. 1, the spacing of stirrups in the left-hand investigated span larger than that in other parts was arranged in order to locate the shear failure in the left-hand investigated span.

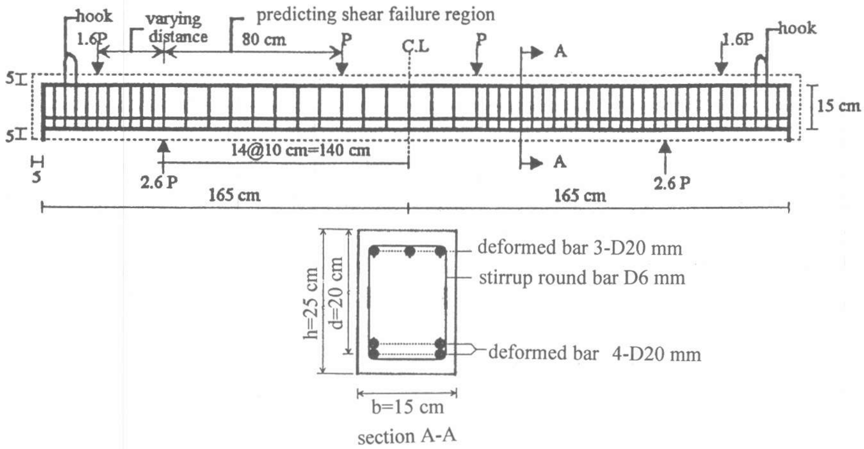

Fig. 1 Beam specimen

2.2 Experimental procedures

The supporting condition for the beam is simple at both ends on 220 cm center to center distance. The concentrated loads were applied by using hydraulic jacks and transferred to different points by means of steel I beams (Fig. 2). During the tests, loads were measured by load cells and strains by strain gauges. It is noted that the strain gauges were attached on both legs of each stirrup in the left-handed investigated span. The positions of inflexure points were varied by changing the distance between a load with the constant value of 1.6 P (where P is the load in the span) and the support in the overhanging portion as indicated in Fig. 1. The test program with different ratios of a_1 /d = 0.5, 1, 2, 3, 4, and 0 (i.e. the case of no inflexure point), where a_1 is the distance of the inflexure point from the nearest support and d is the effective depth of the beam, is shown in Table 1.

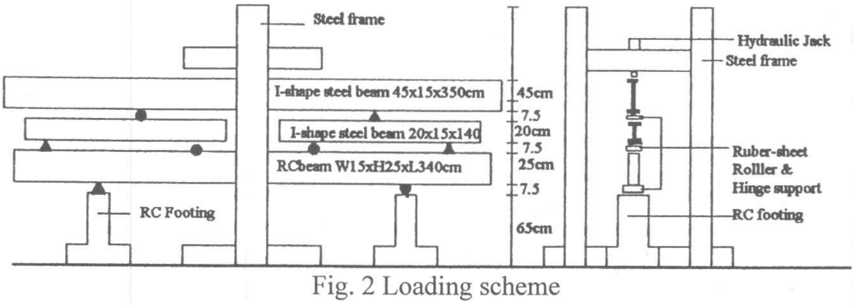

Fig. 2 Loading scheme

Table 1 Test program

Beam No.	a_1/d	Loading pattern	Yielding strength of stirrup (kgf/cm^2)	Compressive strength of concrete (kgf/cm^2)
T-1-1	no inflexure point	two-point loading	2900	302
T-1-2			2900	347
T-2-1	0.5	four-point loading	3300	341
T-2-2	0.5		3300	323
T-3-1	1	four-point loading	3300	304
T-3-2	1		3300	306
T-4	2	four-point loading	2900	323
T-5-1	3	four-point loading	2900	329
T-5-2	3		3300	290
T-6	4	four-point loading	3300	302

3. EXPERIMENTAL RESULTS

3.1 Cracking pattern at the ultimate state

All the beam failed in shear. In the case of the presence of inflexure points in the span such as for the beam T-4 (Fig. 3), as the load increased, the flexural crack occurred first, and then the diagonal shear crack appeared in the shear span. As the external loads were close to an ultimate value, two main diagonal cracks developed, in which the second diagonal cracks (Fig. 3) seemed to have caused the shear failure of the beam. The same tendency was observed in the case of beam without shear reinforcement in the previous study [2].

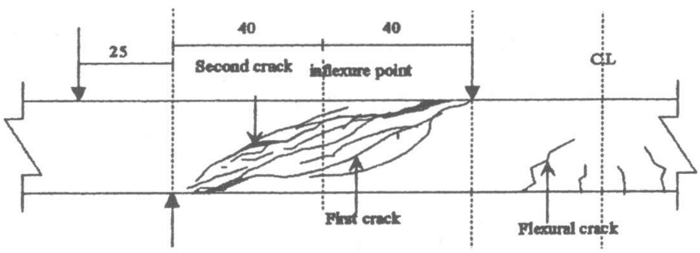

Fig. 3 Cracking pattern at ultimate state for the beam T-4

3.2 Strains in stirrups

Strains were measured during test from both legs of each stirrup. Since almost the same values of the strains of both stirrup legs were obtained, the average values were taken. In Fig. 4, it can be seen from the strains in each stirrup that the stirrup had almost no effects prior to the formation of diagonal cracks, and at the ultimate stage, the yielding of at least one stirrup occurs. The dashed line in Fig. 4 was plotted based on the truss analogy by using an angle between the concrete compression strut and the beam axis (θ=45°). It was found that the dashed line was parallel to most of the strain curves, and hence the truss analogy seemed to be applicable in the beam with the presence of inflexure points.

3.3 Shear capacity of beams

From the experimental results, the beams carried additional loads after the first yielding of stirrups, and when more stirrups yielded, the beams failed at the ultimate load defined by maximum point of the load-deflection curve of the beam. In order to investigate the effects of positions of inflexure points on the ultimate shear capacity, the relation of the

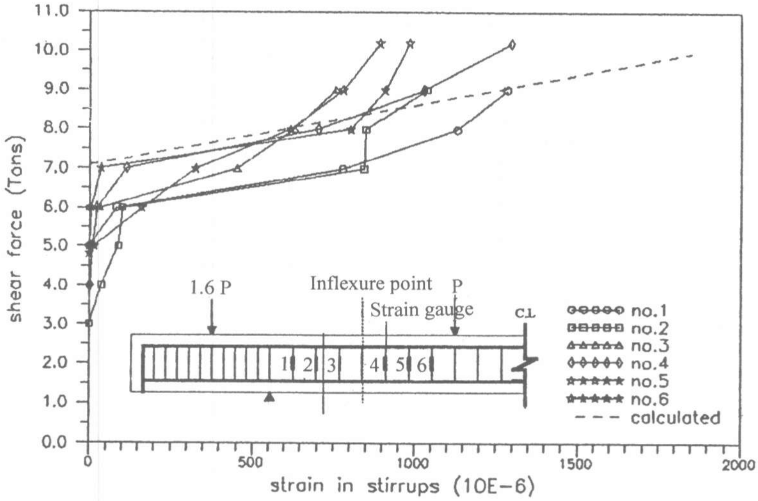

Fig. 4 Strains in stirrups for the Beam T-4

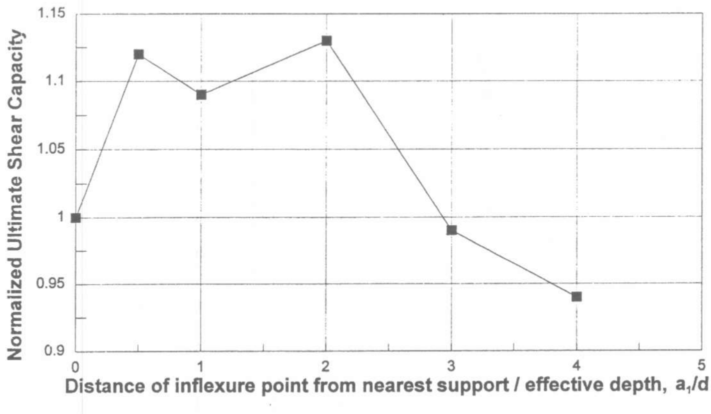

Fig. 5 Relationships between ultimate shear capacity obtained by experiment and equivalent shear span/effective depth ratio

shear capacity obtained by the experiment and the ratio of a_1 /d (defined in Sec. 2.2) was plotted in Fig. 5. It is noted that a vertical axis in Fig. 5 represent the values of ultimate shear capacity normalized by that of the beam without inflexure points (T-1, a_1 /d = 0). It can be seen from Fig. 5 that the ultimate shear capacity was affected by the position of inflexure points in the shear span. The maximum increment of the ultimate shear capacity of the beam with inflexure points with respect to that without inflexure points (a_1 /d = 0) in the range of a_1 /d = 0.5-2 was about 10%. However, in previous study [3], the increment for the case of without shear reinforcement was as much as 70%. The difference of these increment between the cases of the beam with and without shear reinforcement might be explained by the shear resisting mechanisms of the beam with shear reinforcement, i.e., the contribution from concrete and stirrups. In other words, in the beam with shear reinforcement, almost the same values of the ultimate shear force carried by stirrups due to truss mechanisms for the cases of the beams with and without inflexure points reduced the difference of the sum of the ultimate shear force carried by concrete and stirrups of the two cases, although there were larger increment of the ultimate shear force carried by concrete due to effects of inflexure points.

4. ESTIMATION OF ULTIMATE SHEAR CAPACITY

By extending the proposed formula in the previous study on the beam without shear reinforcement [2] to the case of the beam with shear reinforcement, the ultimate shear capacity of the beam with shear reinforcement is predicted by the following equations:

$$V_u = V_{uc} + V_{us} \tag{1}$$

where V_{uc} = the ultimate shear force carried by concrete which is obtained by considering the effects of the positions of inflexure points, and equal to the ultimate shear capacity of the beam without shear reinforcement obtained by the proposed formula in the previous study [2]. It is noted that in the proposed formula of V_{uc}, an assumed fictitious support at the inflexure point divides the shear span "a" into two equivalent shear spans "a_1", "a_2", i.e. the distance of the inflexure points from the nearest support and from the loading point, and V_{uc} is calculated by the JSCE formula [5] with the use of "a_1" or "a_2 for the definition of shear span.

V_{us} = the ultimate shear force carried by stirrups which is expressed by employing the truss analogy as follows:

$$V_{us} = \frac{A_{sw} f_{wy} z}{s}(\cot\theta + \cot\alpha)\sin\alpha \tag{2}$$

in which

α = angle between the shear reinforcement and the beam axis, and α= 90° for the present case of the vertical shear reinforcement; s = spacing of the shear reinforcement; θ = angle between the concrete compression strut and the beam axis in the truss analogy; A_{sw} = area of the shear reinforcement within the spacing s; f_{wy} = yield strength of the shear reinforcement; $z = d/1.5$; d = effective depth of the beam.

It was reported in the past e.g. [3] that the value of θ less than 45° could give more accurate prediction of the ultimate shear capacity compared with the case of θ = 45° . Hence in this study, the different value of θ , i.e. 30° < θ < 45° was employed (see details in [4]), however, in some cases of a_1/d, the calculated values of ultimate shear capacity by using the above value of θ were higher than the experimental results, although the estimation by the different value of θ than 45° seemed to be more accurate than that by the fixed θ = 45°. By considering that the proposed formula of the ultimate shear capacity should give conservative results than the experimental ones, the fixed value of θ = 45° was used in the present study.

By employing Eq.(1) and Eq.(2) with θ = 45°, the predicted values of the ultimate shear capacity for all cases of different a_1/d were computed, and compared with the experimental results in Fig. 6. The predicted values by using the existing different equations [5], [6] were also shown in Fig. 6. From this comparison, the proposed formula gave better prediction than other two formula in JSCE and ACI code which do not take into account of the effects of position of inflexure points. However, there were some discrepancies of about

10 - 20 % between the predicted values by the proposed formula and the experimental ones. This might be due to the lower estimation of the ultimate shear force carried by concrete, i.e. V_{uc} in Eq. (1), and hence, more investigation on V_{uc} related to the effects of positions of inflexure points is needed.

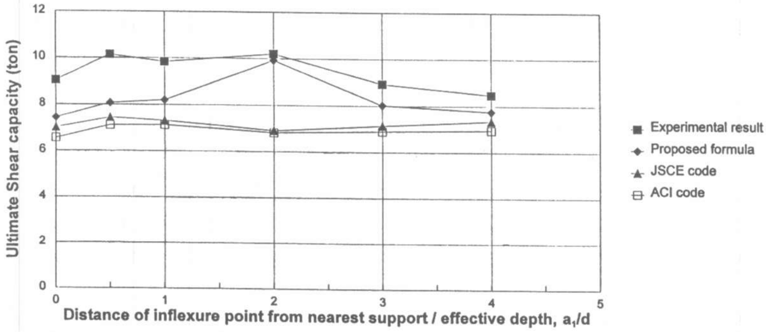

Fig 6 Comparison of ultimate shear capacity

5. CONCLUDING REMARKS

The following concluding remarks may be made in this study:

1. Shear tests on 10 RC beams with the vertical shear reinforcement with different positions of inflexure points were performed to study the effects of inflexure points on the shear capacity of the beam. From the experiment results for different values of a_1 /d = 0,0.5, 1.0, 2.0, 3.0, 4.0, the presence of inflexure points increased the shear capacity as much as 10% in comparison with the beam without inflexure points.
2. By extending the formula proposed in previous study [2] to the case with the shear reinforcement, the formula to estimate the ultimate shear capacity of the beam with shear reinforcement which includes the effects of inflexure points was proposed. There existed better agreement between the results obtained by the proposed formula and the experimental results in comparison with the ACI and JSCE formula. However, more investigations on the ultimate shear force in the proposed formula are needed to improve the accuracy of the formula.

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