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論 文

[2019] Special Finite Elements with Displacement Discontinuity across Internal Interfaces

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1. INTRODUCTION

In the recent year, the modeling of localization problem involving a wide variety of material, and geometrical discontinuities like discrete cracks in concrete, joints in rock and bold-slip layers in reinforced concrete, has become an intriguing issue in the fields of computational failure mechanics. Most of the available finite element techniques for dealing with these discontinuities were developed in the context brittle failure in concrete. As the result, two main development routes, which are closely related, discrete crack model and smeared crack model approach, have been proposed. In the discrete crack approach, crack or other discontinuities are modeled as interelement displacement discontinuities. A stress-displacement relation is specified for the discrete crack;i.e. for the localization summed with no width. When the localization extends through a certain node, this node must be split into two in order to allow the new crack element insertion. The need for repeated changes in the topological connectivity of the mesh is a very serious drawback of this implementation. Despite current attempts to remove the drawbacks of the original representation [1]. Such as the emergence of powerful mesh generators and improved computer hardware, discrete representation of localized failure in three dimensional structure still seems an insurmountable task. Because of these drawbacks, discrete crack models were rapidly replaced by smeared crack models, introduced by Rashid [2]. Smeared crack models are conceptually simple, and can be easily implemented in a general-purpose finite element program. Although these models have been widely used in the analysis of fracture, they still suffer from a number of shortcomings. The main difficulties of spurious mesh-size sensitivity caused by strain softening has been avoided by the introduction of the regularization concepts such as the fracture energy concept [3], rate-dependent model [4], and non-local concept [5]. Although the smeared crack approach gives objective results with regard to mesh size when using non-distorted finite elements, the behavior is not clear when mesh distortions are necessary for modelling requirements; remarkably in these cases a correct energy dissipation cannot be assured, even when the mesh size tends to zero. On the other hand, smeared crack models seem to overestimate the stiffness and strength of structures that exhibits shear-dominated behavior [6,7]. Rots [8] has applied different smeared crack models to cases of localized fracture, and has pointed three problems namely, directional bias, spurious kinematic modes, and stress locking. Regarding these, some formulations aimed at modelling strain localization are found in the literature [9,10,11,12]. In these researches, localization zone of constant width, localized strain modes or discontinuous strain fields are embedded in the finite element. In this paper, we interested to develop new formulations, which are free from the shortcomings of smeared crack and discrete crack model. The study addresses to present an identical model to illustrate interface problems like bond-slip problems, and other localized failure problems within an element. As a main differences between this model and Ortiz's or Dvorkin's model is any interface constitutive model can be adopted to represent the behavior of the internal crack.

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2. SPECIAL FINITE ELEMENTS WITH DISPLACEMENT DISCONTINUITY

2.1. Finite Element Formulation

The incremental formulation for an isoparametric finite element where a strain discontinuity takes place will be developed in this section. We assume that the only nonlinearities in the problem are due to material behavior, and geometrical nonlinearities should be treated in the usual way. Also we assume the general body shown in Figure 1.a. in equilibrium at time (load level) t. At this time the localization phenomenon has been already triggered, and is described in form of an internal interfaces. The configuration shown in Figure 1.b. must satisfy the extended principle of virtual work which was given by Malvern [13], and was used by Dvorkin et al [14], Wu [15] to formulate elements with internal discontinuities.

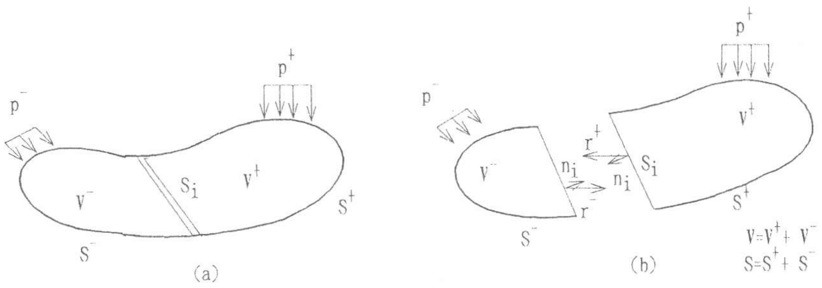

Figure 1. The general body in equilibrium with a displacement discontinuity.

$$\int_{V^-}(\nabla_S \delta u)^T \sigma \, dV^- + \int_{V^+}(\nabla_S \delta u)^T \sigma \, dV^+ - \left(\int_{S^-}\delta u^T p^- dS^- + \int_{S^+}\delta u^T p^+ dS^+\right)$$

$$+ \int_{S_i^-}\delta(u^-)^T r^- dS_i^- + \int_{S_i^+}\delta(u^+)^T r^+ dS_i^+ = 0 \;.\;.\;.\;.\;.\; (1)$$

In which body force is neglected, σ is stress vector, V^+ and V^- are subdomains, $\nabla_S u$ is the strain derived from displacements, **u** is the displacement, $\mathbf{r}^+$ and $\mathbf{r}^-$ are the traction acting on the internal interface, $\mathbf{p}^+$ and $\mathbf{p}^-$ are the prescribed traction.

We now introduce the finite element discretization by considering the following general field approximations.

$$u \approx N_u \bar{u} + N_i \bar{u}_i \;.\;.\;.\;.\;.\; (2)$$

$$u_i = u^+ - u^- \approx (N_i^+ - N_i^-)\,\bar{u}_i \;.\;.\;.\;.\;.\; (3)$$

In which **u** and $\mathbf{u}_i$ is the corresponding nodal displacements, $\mathbf{u}_i$ the displacement in interface, $\mathbf{N}_u$ collects the standard isoparametric, $\mathbf{N}_i$ collects the discontinuous interpolation functions, + and − refer to the positive and negative sides of the interface. Since the displacement field approximation of above equation does not fulfil the C_0 displacement continuity requirement across the interelement boundaries, the resulting finite elements will be incompatible. Constructing compatible finite elements with internal interface is possible by utilizing the extended four field variational statement [16]. With the displacement field approximation of Equation 2,

$$\nabla_S u \approx B_u \bar{u} + B_i \bar{u}_i \quad \ldots\ldots (4)$$

in which B_u is the usual displacement–strain matrix of the finite element, and B_i is a matrix containing the first derivatives of the discontinuous interpolation functions. Now, we consider that δu and δu_i are independent. By discretizing Equation 1 and using the field approximation of Equation 2 and 3 results in the linearized system of Equations:

$$\int_V B_u^T \sigma \, dV = \int_S N_u^T p \, dS \quad ; \quad \int_V B_i^T \sigma \, dV + \int_{S_i} (N_i^+ - N_i^-) \, r_i \, dS = \int_S N_i^T p \, dS \quad \ldots\ldots (5)$$

The above formulation are applicable to the general elements with arbitrary number of nodes and with arbitrary number of internal nodes on the internal interface. If constitutive law and strain compatibility is not enforced in volume V ($V = V^+ + V^-$) a priori, we introduce two additional conditions of global equilibrium:

$$\int_V \delta \epsilon^T (\Sigma - \sigma) \, dV = 0 \quad ; \quad \int_V \delta \sigma^T (\nabla_S u - \epsilon) \, dV = 0 \quad \ldots\ldots (6)$$

where $\boldsymbol{\Sigma}$ is stress vector derived from strain field and $\boldsymbol{\varepsilon}$ strain Vector. Similarly for strain and stress vector, we can consider the following general field approximations:

$$\epsilon \approx N_\epsilon \bar{\epsilon} \quad ; \quad \sigma \approx N_\sigma \bar{\sigma} \quad \ldots\ldots (7)$$

where : $\in$ is the strain mode vector, σ the stress mode vector, $N_\in$ strain interpolation functions, N_σ stress interpolation functions. Then incremental form of above equations can be derived as:

$$\int_V B_u^T N_\sigma \, dV \, \Delta\bar{\sigma} = \int_S N_u^T \Delta p \, dS \quad \ldots\ldots (8)$$

$$\int_V B_i^T N_\sigma \, dV \, \Delta\bar{\sigma} + \int_{S_i} (N_i^+ - N_i^-) \, \Delta r_i \, dS = \int_S N_i^T \Delta p \, dS \quad \ldots\ldots (9)$$

$$\int_V N_\epsilon^T D N_\epsilon \, dV \, \Delta\bar{\epsilon} - \int_V N_\epsilon^T N_\sigma \, dV \, \Delta\bar{\sigma} = 0 \quad \ldots\ldots (10)$$

$$-\int_V N_\sigma^T N_\epsilon \, dV \, \Delta\bar{\epsilon} + \int_V N_\sigma^T B_u \, dV \, \Delta\bar{u} + \int_V N_\sigma^T B_i \, dV \, \Delta\bar{u}_i = 0 \quad \ldots\ldots (11)$$

For the sake of simplicity, a discontinuous interpolation function similar to the Dvorkins's idea [12] is designed for two–dimensional isoparametric element. Decompose the finite element displacement field to a continuous part and a discontinuous part as :

$$u \approx N_u \bar{u} + \Phi_i \bar{u}_i \quad ; \quad \Phi_i = \Phi I \quad \ldots\ldots (12)$$

where u is the nodal displacement vector corresponding to the deformation of the continuum, Φ is considered as a piecewise constant of discontinuous function, I is a (2x2) identity matrix.

$$\Phi = \begin{cases} \omega & \text{in } V^+ \\ \omega - 1 & \text{in } V^- \end{cases} \quad \ldots\ldots (12.a)$$

where $0 \leq \omega \leq 1$.

With the above displacement field approximation the incremental displacement discontinuity is given by

$$u_i = \bar{u}_i \quad \ldots\ldots (12.b)$$

Then by comparing the two assumptions of the finite element displacement field, we obtain

$$\bar{u} = \tilde{u} + P\bar{u}_i \; ; \; N_i = \Phi_i - N_u P \quad \ldots\ldots (13.a)$$

$$P = [\Phi_1 \ldots \Phi_n]^t = [v_1 \; v_2] \quad \ldots\ldots (13.b)$$

in which n = the number of nodes, vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ correspond to nodal displacements due to interface sliding and opening modes, expressed in a local coordinate system attached to the interface. By condensing out ε, σ, and $\mathbf{u}_i$ at the element level, one obtains

$$\bar{K}\bar{u} = \bar{f} \; ; \; \bar{K} = K - KP\,(K_{ii} + P^T KP)^{-1} P^T K \quad \ldots\ldots (14)$$

and

$$\bar{f} = f_u + KP\,(K_{ii} + P^T KP)^{-1}(f_i - P^T f_u) \; ; \; K = K_{\sigma u}^T (K_{\epsilon\sigma}^T K_{\epsilon\epsilon}^{-1} K_{\epsilon\sigma})^{-1} K_{\sigma u} \quad \ldots\ldots (15)$$

in which

$$K_{\sigma u} = \int_V N_\sigma^T B_u \, dV \; ; \; f_u = \int_S N_u^T \Delta p \, dS \quad \ldots\ldots (15.a)$$

$$K_{ii} = \int_{S_i} (N_i^+ - N_i^-)\, \Delta r_i \, dS \; ; \; f_i = \int_S N_i^T \Delta p \, dS \quad \ldots\ldots (15.b)$$

$$K_{\epsilon\epsilon} = \int_V N_\epsilon^T D N_\epsilon \, dV \; ; \; K_{\epsilon\sigma} = \int_V N_\epsilon^T N_\sigma \, dV \quad \ldots\ldots (15.c)$$

$$\Delta r_i = D_i(\Delta u^+ - \Delta u^-) \; ; \; D_i = \begin{bmatrix} E_T & 0 \\ 0 & \beta G \end{bmatrix} \quad \ldots\ldots (15.d)$$

where **D** is the material tangent stiffness, $\mathbf{D}_i$ is the interface tangent stiffness, β is the shear retention factor, and **G** is the elastic shear modulus.

2.2. A Comparison Between Smeared Crack Model and the New Model

It has realized that the elements with smeared crack representation are incapable of representing the sliding mode at the element level. It is shown that the elements developed here are free from the deficiency. By considering Equation 14, which is applicable to a general element, we assume that the interface has reached its final stage, where all the interface stiffness moduli have dropped to zero. In this case, the equivalent stiffness matrix of Equation 15, reduces to

$$\bar{K} = K - KP\,(P^T KP)^{-1} P^T K \quad \ldots\ldots (16)$$

Equation 16 as a subject to the element nodal displacement vector, which corresponds to the two sliding mode v_1 and v_2. Simple calculation shows that :

$$\bar{K}\{v_1 \quad v_2\} = \bar{K}P = KP - KP = 0 \; . \; . \; . \; . \; . \; (17)$$

which indicates that the element is capable of properly representing different deformation of the interface at the element level.

3. NUMERICAL EXAMPLES

We consider a typical problem whereby the crack propagates through the mesh in a zig-zag manner. It concerns a Crack-Line-Wedge-Loaded Double-Cantilever-Beam (CLWL-DCB) which has been tested by Kobayashi et.al.[17] (Figure 2). The specimen of 50.8 mm thickness is assumed to be in a state of plane stress, and is subjected to a wedge load F_1 as well as a diagonal compression load F_2. The ratio of the diagonal force to the wedge force is kept approximately constant at 0.6 until a predetermined diagonal force of 3.78 kN is reached, whereafter the diagonal force is kept constant and only the wedge force alters.

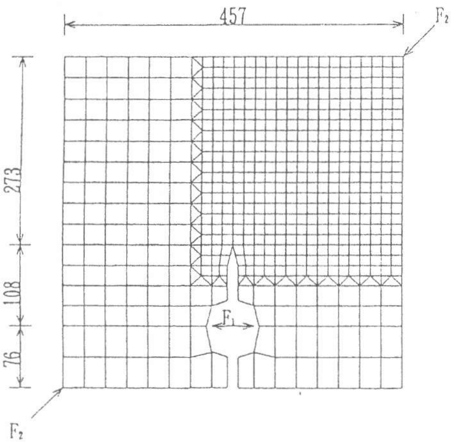

Figure 2. Four-node quadrilateral finite element idealization of notched CLWL-DCB specimen, dimension in mm

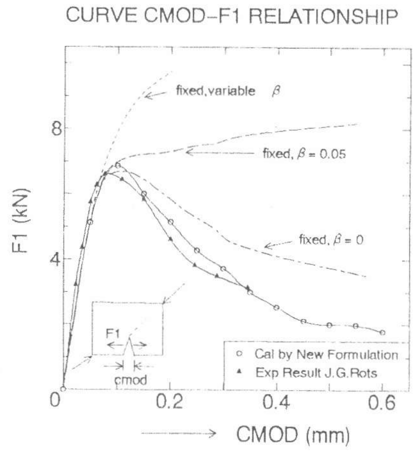

Figure 3. Result comparisons

Here, we used the finite element lay-out constant as we would only use the mesh of Figure 2, with four-node quadrilaterals, and investigate the effect of constitutive formulations. Although the loading consists of a wedge load and a diagonal compression load, the crack tip is in a state of tension-tension and propagates primarily in mode I. For nonlinear inelastic interfaces, the interface tangent stiffness identical with Rots study [8] is adopted, i.e. a linear stress displacement incremental constitutive relation corresponding to Mode I blunt fracture adjusted to a given fracture energy is used for crack opening. Figure 3 presents the Load-Crack Mouth Opening Displacement (CMOD) response by using new formulation which was compared with results of experimental response and the responses calculated by smeared crack model [8]. The smeared crack model have been focused on the shear stiffness reduction factor or the shear retention function β, while β was varied; β = 0;0.05;variable. Figure 3 shows that all smeared crack results are too stiff. Fixed cracks with significant shear retention fail in this respect. The latter statement already holds for a β value of 0.05, which is low compared to what is commonly used in Engineering practice. The result for β = 0.05 shows that the limit point has vanished while the response continues to shelve instead of softening. As an effects of stress locking and spurious kinematic mode in the analysis are two possibilities: the stress at the first element will either exceed the tensile strength and start softening, or it will not. Our prediction is also closed to the response calculated by discrete crack models.

4. CONCLUSION

In this paper we develop an identical formulation for localization and other local discontinuous problems, where for the discontinuous interface a local stress-displacement constitutive relation is used. The new formulation keeps objectivity with regard to regular mesh refinements.

Although the new formulation introduces a non symmetric consistent stiffness matrix, only the symmetric part of the consistent stiffness matrix can be used with a limited efficiency decrease. This makes the finite element with embedded discontinuous interface easy to implement in standard non linear finite element codes, and therefore all the existing finite element libraries and constitutive models for the material outside the discontinuous interface can be readily used in combination with them.

From comparison between smeared crack concept model, it shows that smeared crack model is too stiff, and new formulation method is almost same with the experimental result or the calculated result by discrete crack model which has been done by Rots [8]. Recently, the application of the proposed model for solving the behavior of bond-slip interfaces between steel and concrete is in progress.

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