

[2013] An Analytical Study on Shear Walls Subjected to Biaxial Loading

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1. INTRODUCTION

The last decade witnessed rapid advances in the use of finite elements methods (FEM) for the analysis of reinforced concrete(RC) structures. Due to its highly complex material behavior, the analysis of RC structures by the finite element method is still under improvement.

Recent researches reflect this improvement with regard to the RC plates or shells. Massicote et al. [1] analyzed panels subjected to axial and lateral loads by implementing the three-dimensional degenerate plate element. Many constitutive models and analytical procedures have been reported in many papers; Shenglin and Cheung, [2], Xiang et al. [3], Hu and Schnobrich [4]. However most formulations were based on specialized elements. Besides, most concerns were concentrated on the analysis of elements loaded biaxially at the same time without paying enough attention to the mutual effects of sequential loading.

In a previous paper [5], the authors proposed a simplified procedure to analyze thin plates that are subjected to biaxial loading. The procedure was based on the implementation of both membrane and Mindlin elements. However, further improvement was deemed necessary to analyze thick plates as well. This paper presents the improved process that describes the behavior of such members when subjected to combined loads applied in a successive manner. The analytical results are then compared to experimental results on shear walls which have been subjected to biaxial loading conditions. This study used the heterosis degenerate element for analyzing thick plates, and as a next step, the same element will be used for analyzing general shell structures as well.

2. FINITE ELEMENT MODEL

To represent the behavior of such panels adequately, the following requirement are essential in modeling the structure. The element should enable the modeling of both thin and thick plates and it must have the capability of handling large deflections and second-order effects. Besides, the material model must describe

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the RC behavior in plane stress conditions.

Two basic assumptions were adopted in the process: Firstly the normals to the middle surface remain practically straight after deformation (Fig.1). Secondly, the

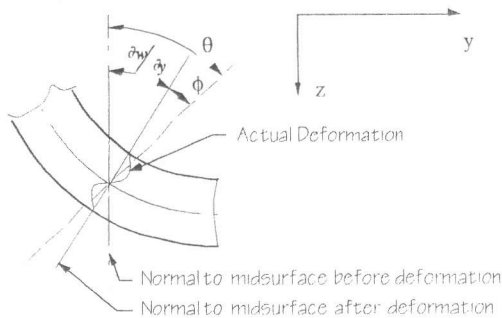


Fig.1 Deformation of Plate

strain energy corresponding to stresses perpendicular to the middle surface is disregarded, i.e. the stress component normal to the plate mid-surface is constrained to be zero in the constitutive equations.

The element used in the analysis is the quadratic heterosis element (developed originally for plates [6]) with nine nodes and 42 degrees of freedom. This element was degenerated from the quadratic solid three-dimensional element as shown in Fig.2. This element is manageable when applying smeared rotating cracks approach, besides, it exhibits many advantages [10].

Elements are divided into layers allowing a discretized variation of material properties and nonlinearities as loading progress, yet retains the limited degrees of freedom of two dimensional approach. Serendipity shape functions for translational degrees of freedom and Lagrangian shape functions for rotational degrees of freedom were considered. The major advantages of such choice is allowing variable thickness and curved sides.

Both large displacement and material nonlinearity are considered in the process. Direct solution is adopted where at each iteration the total displacements δ_i are determined by the full load applied F ;

$$F = K_{i-1} \delta_i \dots \dots (1)$$

3. CONSTITUTIVE MODEL

The constitutive model is based on the theory proposed by Vecchio and

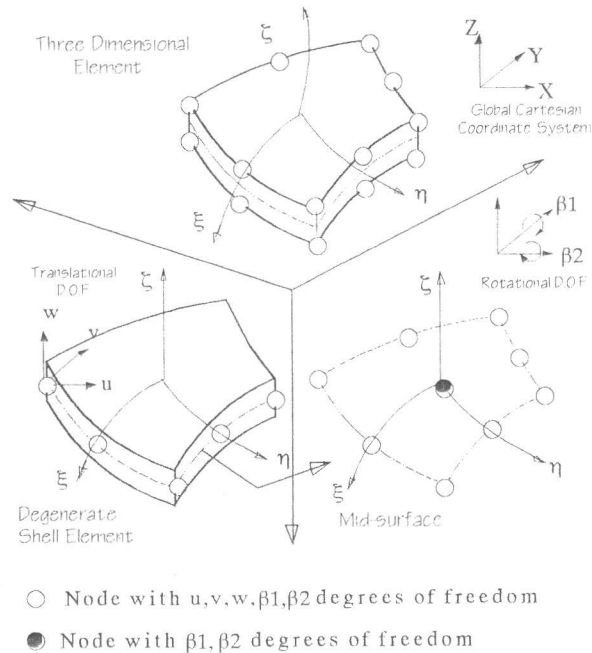


Fig.2 The Degenerate Element

Collins [7]. The model considers that the degradation of the concrete compressive strength is due to transverse tensile strain after cracking. Also it assumes the directions of principal stresses and strains in concrete to be identical.

Smearred Cracks approach is adopted and rotating cracks are assumed to form following the principal strains orientation. The concrete material stiffness matrix D_c is evaluated using the proposed theory in the principal strain direction then transformed to the local xy system by

$$D'_c = T^T D_c T \dots\dots\dots (2)$$

where T is the strain transformation matrix.

Reinforcement steel in the xy plane is treated as explained in [5]. The strains of the transverse steel reinforcement are calculated by using appropriate constitutive relation. The material stiffness D_{s-o} in that direction is calculated as;

$$D_{s-o} = \begin{bmatrix} E_{s-o} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \dots\dots\dots (3)$$

where E_{s-o} is the stiffness in the transverse direction.

The calculated stiffness matrix is translated to the local reference xyz system, condensed by appropriate procedures, and then added to the concrete stiffness in the reference xy by;

$$D = D_c + \sum D_{sj} \dots\dots\dots (4)$$

Poisson effect was assumed to be effective till cracking of concrete. Expansion and confinement effect were taken into account when calculating the Poisson ratio [8].

The analysis is carried out in the same order of loading -such as the case of the example examined in this paper-. In this sequence, many variables such as cracked or crushed Gaussian points are carried over from one loading stage to another.

4. COMPARISON WITH EXPERIMENTAL RESULTS

The analytical procedure was applied on tests conducted by Umemura and Aoyama [9] in which four test specimens were subjected to both constant pure flexure (out-of-plane) and then to in-plane force.

The test specimen is 150 mm thick by 500 mm height by 2200 mm length with reinforcement distributed as in Fig.3-a. Each wall was reinforced by two layers of deformed bars placed in orthogonal directions, with the reinforcement ratios for the top and bottom layers equal.

Out-of-plane reinforcement were provided by D13 stirrups. Based on the difference of the design concrete strength, the test specimens were grouped into two separate groups (No.1, No.2 and No.3, No.4).

Material properties are shown in Table 1 where σ_c and E are compressive strength and Young modulus of concrete respectively, σ_y is yielding strength of steel. Shape and reinforcement of one-half of the test specimen, deflection shapes loading and support conditions are shown in Fig.3-b.

In the in-plane loading direction, the test specimen was subjected to anti-symmetrical conditions (loaded at two points in opposite directions). As for the out-of-plane direction, it was loaded at two points in one direction. The jack applying the in-plane force was a reversible of 200 ton capacity, and the jack applying load in the out-of-plane direction was of 20 ton capacity. The constant out-of-plane loading varied (as shown in Table 2) from 0, 3, 6 and 8 tons for test specimen No.1, No.2, No.3 and No.4 respectively.

Dial gauges were used to measure displacements. In-plane displacements (δ_{in}) were measured at one-third the test specimen. Out-of-plane displacements (δ_{out}) were measured at the mid and ends of the test specimen.

At first the out-of-plane load was applied to the pre-determined level then the in-plane loading was applied in successive reversals. The first cycle had a peak displacement of 1/250 (0.24 cm). After that, loading was reversed to get the ultimate resistance. At the third cycle the test specimen was loaded

Table 1 Material Properties

	No.1, No.2	No.3, No.4
Concrete	$\sigma_c=249$ $E=2.34 \times 10^5$	$\sigma_c=327$ $E=2.69 \times 10^5$
D13	$\sigma_y=3570$	
D19	$\sigma_y=3710$	
D25	$\sigma_y=3710$	

Unit; kgf/cm²

footnotes: σ_c : Compressive Strength of Concrete
 E : Young Modulus of Concrete
 σ_y : Yielding Strength of Steel

Table 2 Applied Out-of-plane Loads

	No.1	No.2	No.3	No.4
Applied Out-of-plane Load	0	3	6	8

Unit; tf

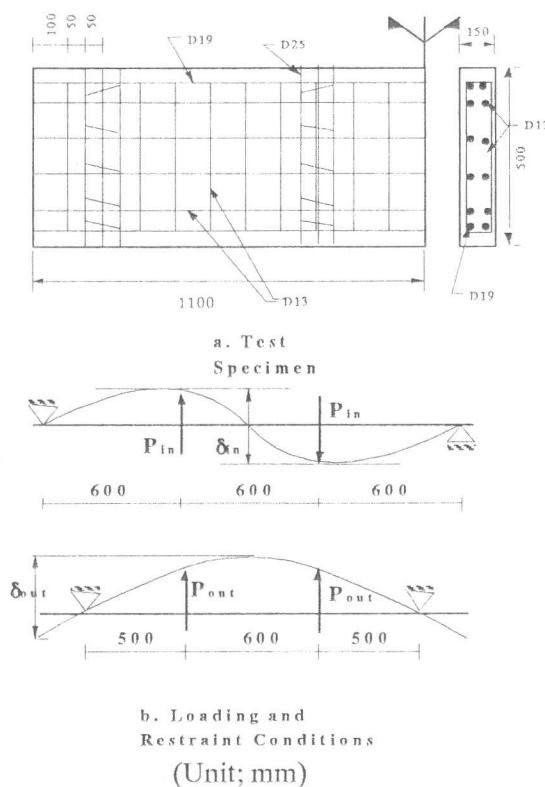


Fig.3 The Test Specimen

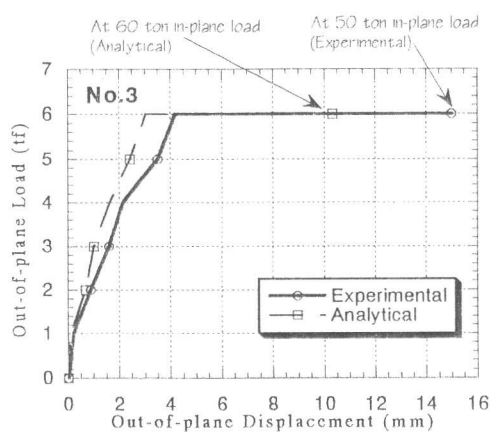
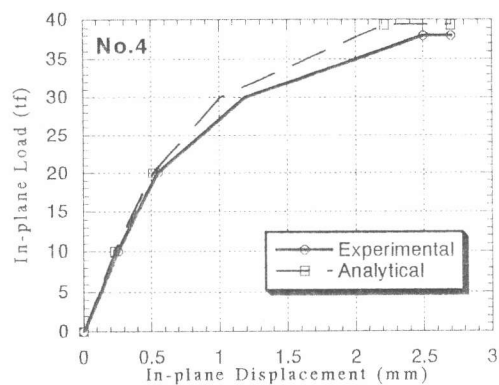
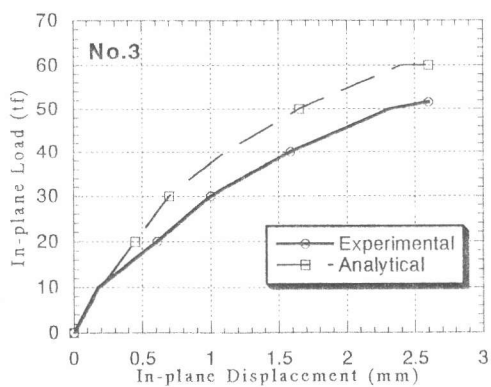
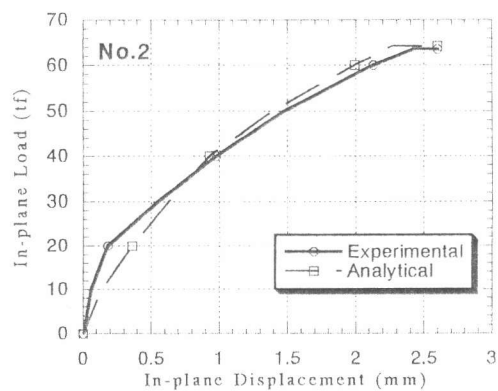
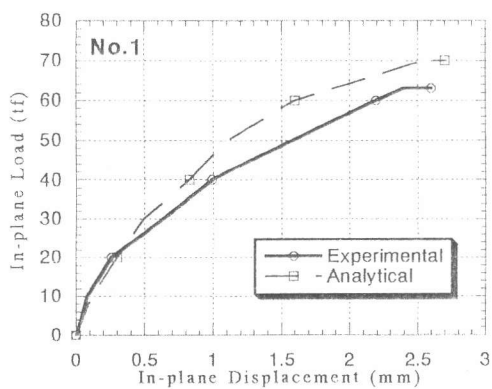


Fig. 4 Load-displacement Results

a. In-plane Load vs.
In-plane Displacement

b. Out-of-plane Load vs.
Out-of-plane Displacement

till collapse.

To model the reinforcement and specimen details, a relatively fine mesh of four by eight element grid was necessary for a half of the test specimen (Fig.5). Each element was discretized into 12 layers.

The comparison between the experimental and analytical results was carried out with regards to the peak loads. It was also carried out on the out-of-plane deflection (only for No.3 where data is available).

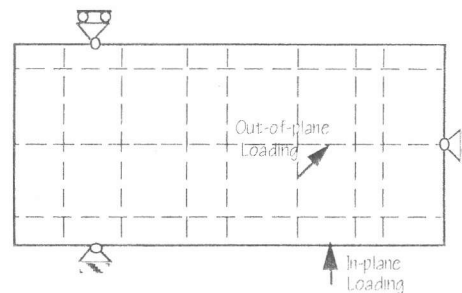


Fig. 5 Finite Element Model
(xy-Direction)

Fig.4-a shows the plots of the experimental and analytical in-plane load versus displacement. It can be seen that the analytical results simulate the experimental ones reasonably. Fig.4-b shows the plot of the out-of-plane load versus the out-of-plane displacement both experimentally and analytically. A good agreement can be also observed with regard to the out-of-plane load-displacement curves. Ignoring the loading sequence effect in the analysis would have resulted in less displacements than those shown in the figures and less accurate results.

5. CONCLUSIONS

A procedure to analyze reinforced concrete plate elements under combined loading conditions has been presented. The procedure is based on the implementation of heterosis element using the modified compression field theory. It takes into account the loading sequence effect further improving the procedure accuracy. The procedure is general and can be applied on both thick and thin plates. A good representation of the plate behavior could be obtained with respect to in-plane and out-of-plane load-displacement curves. Further comparative study by using the generalized shell element is a prospective one.

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