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論 文

[2218] BEM Analysis of Mixed-Mode Crack Propagation in Center-Notched Concrete Beams

Ali CHAHROUR[*1], *Shinichi FUKUCHI*[*2], *Masayasu OHTSU*[*3] and *Yuichi TOMODA*[*4]

1. INTRODUCTION

The applicability of fracture mechanics principles to analyze crack initiation and propagation in concrete under tensile mode-I loading has encouraged researchers to try applying them for mixed-mode fracture investigation. In actual concrete structures, cracks propagate under the combined action of both tensile and shear fractures. In fracture mechanics terms, these are referred to as mode-I and mode-II fracture types, respectively.

Many researchers have tried analyzing crack growth in concrete structures employing the finite element method (FEM). In this respect, the task could be very tedious and cumbersome, for FEM techniques require remeshing of the whole domain to simulate crack propagation. Hence, the choice of the boundary element method (BEM) to simulate mixed-mode fracture is quite reasonable[1]. Compared to FEM, BEM needs fewer input data and allows even simpler automatic remeshing of the boundary to accomodate crack propagation.

In the present paper, a two-domain boundary element method approach is adopted to analyze mixed-mode crack propagation in center-notched concrete beams subjected to one-point and two-point loadings. In the procedure, stress intensity factors at a notch tip are computed from relative displacements on the crack-tip boundary element. The direction of crack propagation is determined, based on the criterion of maximum circumferential stress of linear elastic fracture mechanics (LEFM).

The analyzed crack trajectories, load-CMOD curves and relative values of the stress intensity factors K_I and K_{II} are discussed for the cases of center-notched concrete beams with 20, 30 and 40 mm notch lengths. Results reveal the ability of the technique, introduced in a authors' previous work[2], with some modifications to simulate crack growth.

2. TWO-DOMAIN BEM FORMULATION

Fig. 1 shows the case of two elastic domains D_1 and D_2 joined along a common interface S_C. S_1 and S_2 represent the external boundaries of D_1 and D_2, respectively. Hence, $S_{1T}=S_1+S_C$ and $S_{2T}=S_2+S_C$ designate the boundaries of D_1 and D_2, respectively.

The boundary integral equation for each region α is given as follows[3]:

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*1 Graduate School of Kumamoto University
*2 Nippon Koei Consultant Corporation
*3 Professor, Department of Civil & Environmental Engineering, Kumamoto University
*4 Department of Civil & Environmental Engineering, Kumamoto University

$$C_{ij}^{\alpha}u_i^{\alpha}(x) + \int_{S^{\alpha}} T_{ij}^{\alpha}(x,y)u_j^{\alpha}(y)dS = \int_{S^{\alpha}} G_{ij}^{\alpha}(x,y)t_j^{\alpha}(y)dS, \tag{1}$$

where S^{α} is equal to S_{1T} for D_1, and S_{2T} for D_2.

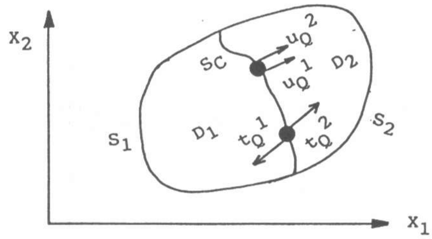


Fig. 1 Subdivision of computational domain

The boundary conditions are those imposed separately on S_1 and S_2, while the interface conditions on S_C are given by,

$$u_j^1(Q) = u_j^2(Q)$$
$$t_j^1(Q) = -t_j^2(Q). \qquad (Q \in S_C) \tag{2}$$

A constant element is adopted for traction and a linear element is assigned for the displacement on the boundary elements[2]. The analysis is carried out for the plane-strain case. Discretizing the two domains and the interface boundary, and carrying out the numerical integration over the elements using 6-point Gaussian integration, Eq. 1 can be written in the following form,

$$[H_{ij}]\{u_j(y_k)\} = [G_{ij}]\{t_j(yy_k)\}. \tag{3}$$

Tractions on the elements are converted into nodal forces by introducing shape functions to the above equation. Making use of Eqs. 2, the matrix Eqs. 3 for (α =1,2) are coupled together, and the resulting equation is solved for the nodal forces and displacements.

3. ANALYTICAL MODELS AND ANALYSIS PROCEDURE

The analytical models for the analysis of mixed-mode fracture in center-notched concrete beams subjected to one-point and two-point loadings are given in Figs. 2 and 3,

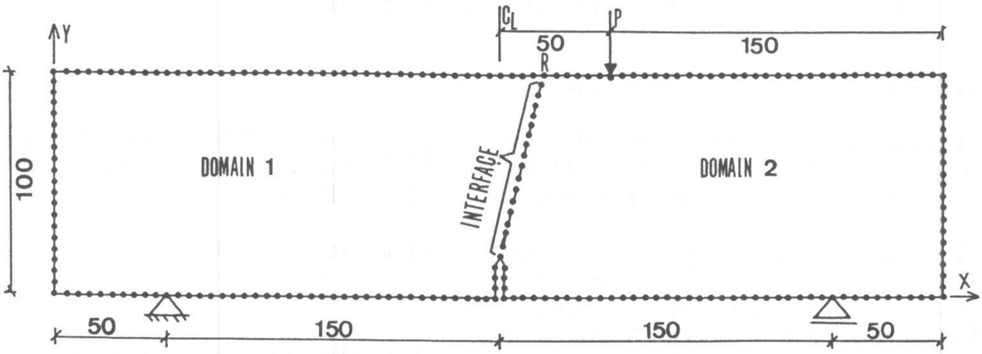


Fig. 2 Two domain BEM model (one-point loading)

respectively, where all dimensions of the beams are given in mm.

Double nodes are assigned along the notch, and the broken line with nodes represents the stitching interface. It joins the notch tip to point R on the top surface of the beam.

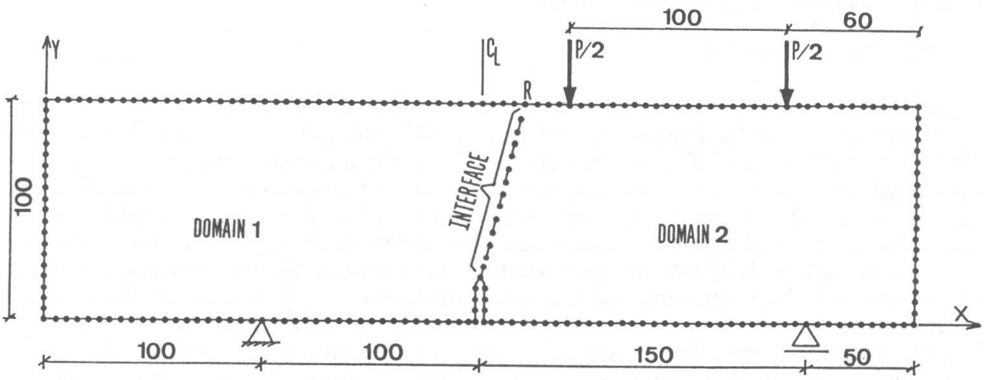


Fig. 3 Two–domain BEM model (Two–point loading)

When a crack propagates, the node at the tip is seperated into two nodes, creating two new crack tip elements. This results in the increase of the total number of nodes. A new stitching boundary is defined as the straight line joining the new crack tip elements, defined based on the crack direction and increment, to point R. Automatic remeshing is accomodated to take care of the newly created stress–free crack tip elements. The same element length of 5mm was considered for domains 1 and 2, and for the crack increment during crack propagation.

The stress intensity factors at the crack tip, K_I and K_{II}, were determined by Smith's one point formulae[4]. The direction of crack growth is determined based on the criterion of the maximum circumferential stress given by Eq. 4, and Eq. 5 states the adopted fracture criterion for crack propagation,

$$K_I^* \sin \phi + K_{II}^*(3\cos \phi - 1) = 0, \tag{4}$$

$$\cos(\phi/2)[K_I^* \cos^2(\phi/2) - 3K_{II}^* \sin\phi/2] \geq 1, \tag{5}$$

where $K_I^* = K_I/K_{IC}$ and $K_{II}^* = K_{II}/K_{IC}$. K_{IC} is the fracture toughness of concrete considered a material property. ϕ is taken to be the angle corresponding to the bigger maximum tensile stress as shown in Fig. 4.

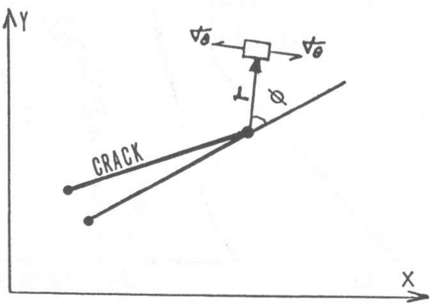


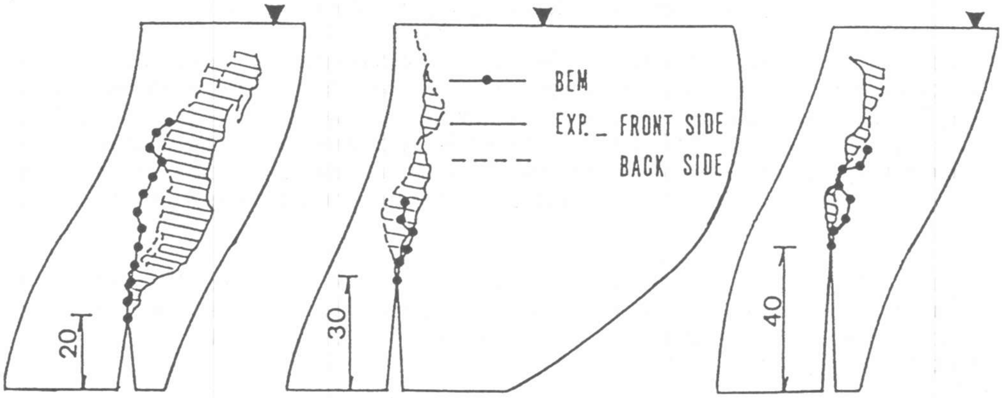
Fig. 4 Crack growth direction

4. ANALYTICAL RESULTS AND DISCUSSION

Mechanical properties of concrete were as follows: compressive strength was 36.3 Mpa, tensile strength was 3.2 Mpa, Young's modulus E was 29.4 Gpa, Poisson's ratio ν was 0.21, and fracture toughness K_{IC} was 0.49 $Mpa.m^{1/2}$

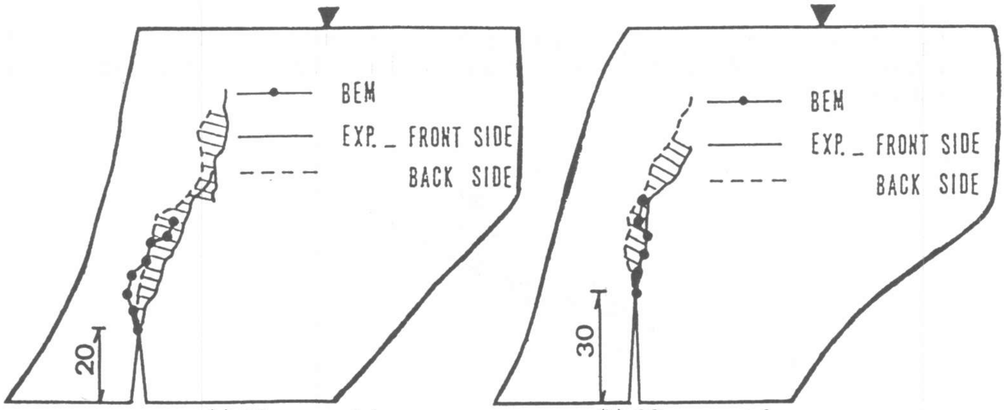
4.1 CRACK TRAJECTORIES

Figs. 5(a), (b), and (c) show the analytical and experimental results for crack trajectories of mixed–mode fracture in center–notched beams subjected to two–point loading with the notch length of 20, 30, and 40 mm, respectively. Similarly, the crack trajectories of center–notched concrete beams subjected to one–point loading are given in Figs. 6(a) and (b). The shaded areas reveal the crack front across the beam width. The simulated BEM results showed good agreement with the actual crack trajectories. In relation to the numerical results, two notes are highlighted here that need careful attention[5]. The first point is that it seems important to keep the same mesh division on the boundary surfaces of the two domains and along the interface. The second point to note is related to the choice of point R on the top surface of the beam. In this respect, experience showed that the interface should end up at a point on the top surface somewhere near the actual termination point of the crack.



(a) 20 mm notch (b) 30 mm notch (c) 40 mm notch

Fig. 5 Actual and simulated crack trajectories (two–point loading)



(a) 20 mm notch (b) 30 mm notch

Fig. 6 Actual and simulated crack trajectories (one–point loading)

4.2 LOAD–CMOD CURVES

Load–CMOD curves for center–notched beams of 20 mm notch depth subjected to two–point and one–point loadings are shown in Figs. 7 and 8, respectively. The analytical curve reproduces the essential feature of the experimental load–CMOD curve, even though an abrupt decrease in the analytical load is observed. The peak load was overestimated in the two–point loading case and underestimated for the one–point loading case. Loading configuration and notch length seem to be responsible for these discrepancies.

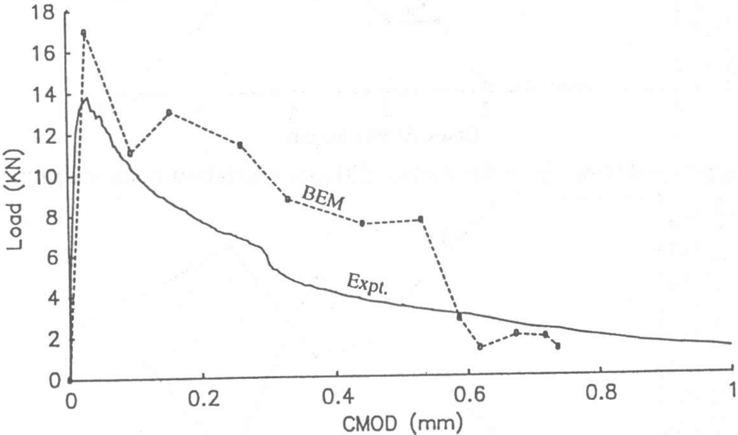


Fig. 7 Load–CMOD curve (two–point loading/20 mm notch)

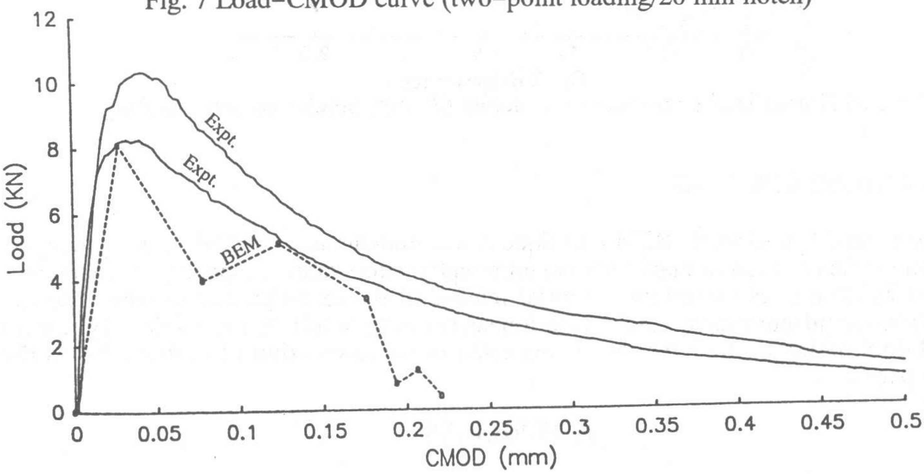


Fig. 8 Load–CMOD curve (one–point loading/20 mm notch)

4.3 ANALYSIS OF CRACK PROPAGATION MECHANISM

Normalized stress intensity factors K_I^* and $|K_{II}^*|$ values versus crack extension for 20 mm notched beams subjected to two–point and one–point loadings are shown in Figs. 9 and 10, respectively. Absolute values of K_{II}^* are considered because negative values were obtained at several stages in the analysis. The results reveal that crack starts from the notch tip with K_I^* almost equal to unity. This indicates that the crack starts with almost pure mode–I fracture. As the crack propagates, K_{II}^* increases and gradually mode–II fracture becomes more dominant. Therefore, the beam ends up failing in shear with $|K_{II}^*|$ approaching unity. The analysis was terminated when the mode–I stress intensity factor becomes negative, which corresponds to the inward overlapping of the opening displacements at the crack tip element.

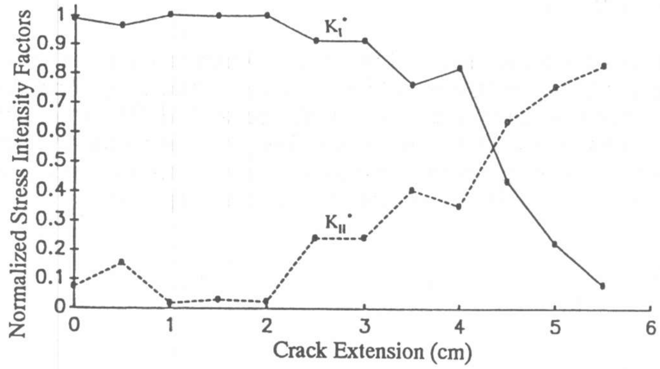


Fig. 9 Normalized stress intensity factors (20 mm notch/two point loading)

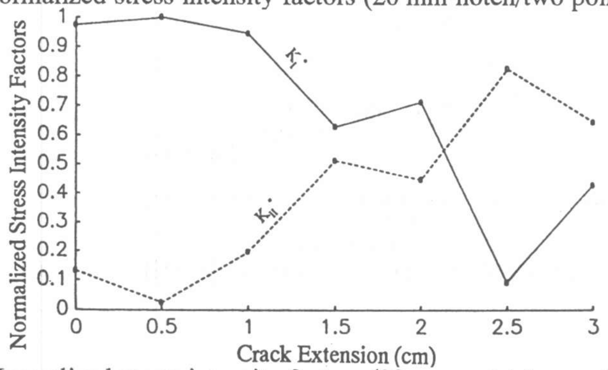


Fig. 10 Normalized stress intensity factors (20 mm notch/one point loading)

4. CONCLUDING REMARKS

The presented two-domain BEM technique could simulate mixed-mode crack propagation based on the criterion of maximum circumferential stress quite reasonably. The simulated load-CMOD curves reproduce essential feature of the experimental curves. For the given loading configurations, crack initiation seems to be solely of the mode-I type with mode-II dominating at the later stages, resulting in the interaction of both modes as the crack propagates.

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