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論 文

[2175] BEM Analysis of Mixed-Mode Fracture

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1. INTRODUCTION

Crack propagation in concrete structures normally takes place under combined action of both tensile (mode–I) and shear (mode–II) types of loading. Induced mixed–mode fracture complicates the problem for determining crack trajectory and crack instability. Accordingly, the development of a numerical analysis technique for mixed–mode cracking is of utmost importance and a top priority in research on fracture mechanics of concrete.

In the present paper, a multi–domain BEM (boundary element method) approach is adopted to analyze mixed–mode cracking in concrete based on linear elastic fracture mechanics (LEFM). The choice of LEFM stems from the fact that nonlinear models for mixed–mode fracture analysis so far have not been fully established yet. Mixed–mode fractures in center–notched concrete beams subjected to anti–symmetric loading and of anchor bolts embedded in concrete blocks are analyzed. Crack propagation is modeled by creating new boundaries in the numerical analysis. In the procedure, stress intensity factors at a notch tip are computed from displacements on the crack tip boundary element. Based on the computation of the direction of crack propagation, a new boundary is created in the multi–domain, which consists of two domains stitched at the interface boundary. Hence, automatic remeshing of the boundary to accomodate the increment of crack growth is carried out, and the prediction of crack trajectories is performed by just creating new boundaries. Thus, crack trajectory based on the criterion of maximum circumferential tensile stress is investigated.

2. BEM FORMULATION AND ANALYTICAL PROCEDURE

The boundary integral equation is given, as follows:

$$Cu_i(x) = \oint_S [\, G_{ik}(x,y)t_k(y) - T_{ik}(x,y)u_k(y) \,]dS \qquad (1)$$

where C is the configuration coefficient determined by eliminating rigid motion in Eq. 1. $G_{ik}(x,y)$ is the fundamental singular solution for the 2–D (two dimensional) linear elastic field of the plane strain state, and $T_{ik}(x,y)$ is the associated traction solution.

The boundary of the specimen is discretized into elements L_j (j=1,...,N), and thus Eq. 1 becomes,

$$Cu_i(x) + \sum_{j=1}^{N} \oint_{L_j} T_{ik}(x,y)u_k(y)dS = \sum_{j=1}^{N} \oint_{L_j} G_{ik}(x,y)t_k(y)dS \qquad (2)$$

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To analyze crack propagation in an arbitrary direction, a multi-domain approach is employed [1]. Here, a constant element is adopted for traction and a linear element is assigned for displacement on the element[2],

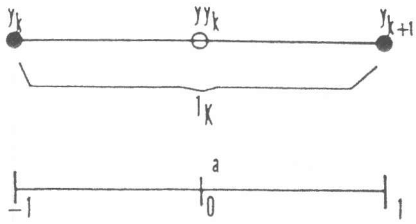

Fig. 1 Linear boundary element

$$\{t(a)\} = t_j(yy_k) = \text{Constant}$$
$$\{u(a)\} = (1-a)u_j(y_k)/2 + (1+a)u_j(y_{k+1})/2 \quad (3)$$

where a is the variable of the local coordinates as shown in Fig. 1. The integration of each element is carried out by the Gaussian 4-point scheme. Taking account of the diagonal terms in Eq. 1, we obtain the matrix form,

$$[H_{ij}]\{u_j(y_k)\} = [G_{ij}]\{t_j(yy_k)\} \quad (4)$$

To make the numerical scheme comparable to the conventional FEM code, Eq. 4 is converted into the equation, not of traction on the element, but of nodal forces. Eventually, the following equation is obtained,

$$N_{km}[G^{-1}{}_{ml}H_{lj}]\{u_j\} = \{f_k\} \quad (5)$$

where N_{km} is the shape function for obtaining nodal forces from tractions. Eq. 5 is of the same stiffness matrix as in FEM, and thus is readily coupled with the other domain.

As an example, a model for the analysis of a center-notched concrete beam subjected to anti-symmetric loading is given in Fig. 2, where double nodes are assigned along the stitching boundary. When a crack propagates, a node at the tip is separated into two nodes. It implies addition of a new node on the stress-free crack surface, resulting in the increase of the total number of nodes.

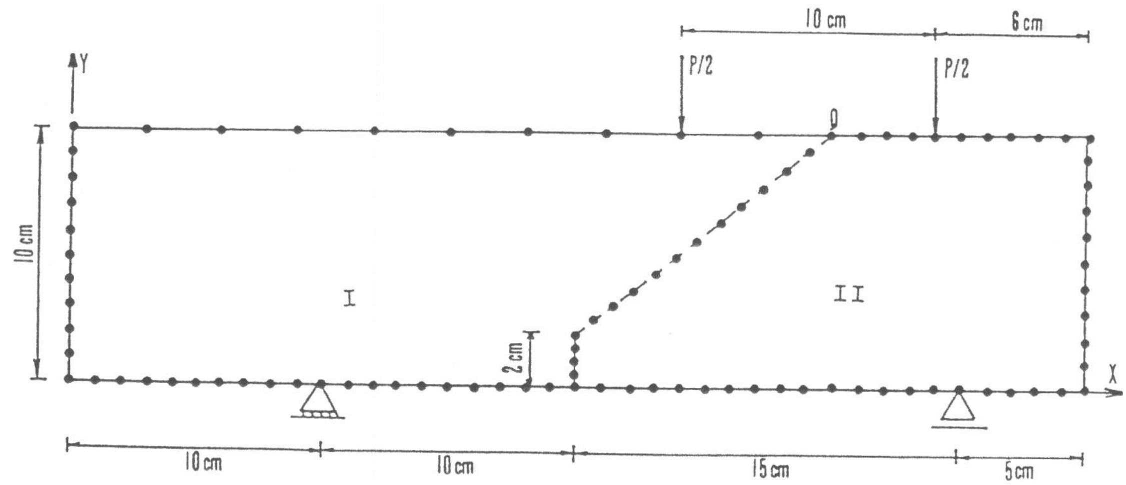

Fig. 2 Two-domain BEM model (center-notched beam)

For the case of mixed-mode fracture in the pull-out test of anchor bolt embedded in a concrete block, two models were considered corresponding to different locations of the reacting force and different sizes of the concrete blocks. In both models, the stitching boundary was taken to be a straight line joining the crack tip to the node where the reaction is supported. Needless to say, the crack tip changes location with crack propagation in connection with the new direction along the crack extension.

The stress intensity factors at the crack tip were determined by Smith's one point formulae[3]. K_I and K_{II} are determined by using displacement u_i in the X-direction and displacement v_i in the Y-direction, shown in Fig. 3,

$$K_I = (2\pi /L)^{1/2}E[\, v_i(B) - v_i(A)\,]/4(1 - \nu^2) \qquad (6)$$

$$K_{II} = (2\pi /L)^{1/2}E[\, u_i(B) - u_i(A)\,]/4(1 - \nu^2) \qquad (7)$$

Here L is the distance from the crack tip.

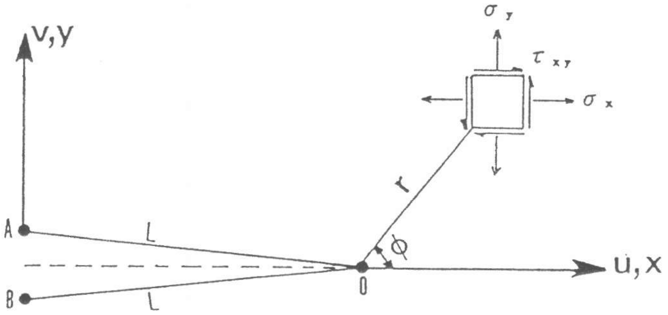

Fig. 3 Crack tip element

The direction of crack growth is determined based on the criterion of maximum circumferential stress given by Eq. 8, and Eq. 9 states the adopted fracture criterion for crack propagation [4],

$$K_I \sin \phi + K_{II}(3\cos \phi - 1) = 0, \qquad (8)$$

$$\cos(\phi/2)[K_I \cos^2(\phi/2) - 3K_{II}\sin\phi/2] \geqq K_{IC} \qquad (9)$$

where the stress intensity factors K_I and K_{II} are computed from Eqs. 6 and 7, by substituting relative displacements at the nodes on the crack tip element. K_{IC} is the fracture toughness of concrete considered a material property. From Eq. 8, the direction of crack propagation is given. Here, ϕ is taken to be the angle corresponding to the bigger maximum tensile strength as shown in Fig. 4.

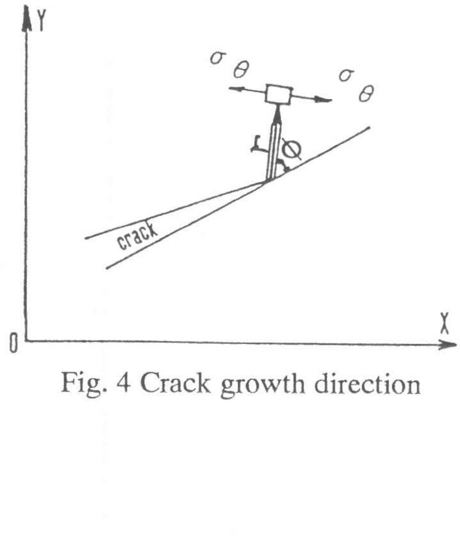

Fig. 4 Crack growth direction

In the case of the center–notched beam, the crack propagates by 5mm length at every step, and a new coplanar stress–free boundary is added in front of the crack tip. The stitching boundary is created by connecting the crack tip to node Q on the top surface of the beam as shown in Fig. 2. In the case of the anchor bolt problems, the crack propagates 1cm at a time, because the concrete block is fairly big compared to the beam.

3. ANALYTICAL RESULTS AND DISCUSSION

Mechanical properties of concrete were as follows: compressive strength was 36.3 Mpa, tensile strength was 3.2 Mpa, Young's modulus E was 29.4 Gpa, Poisson's ratio ν was 0.21, and fracture toughness K_{IC} was 0.49 Mpa.m$^{1/2}$

An analytical result obtained for the case of a 2cm notched–beam undergoing mixed–mode fracture is shown in Fig. 5. Crack trajectories are compared between an analytical result and an experimental result[2]. Since the crack elements of 5 mm length were employed, the determined profile of crack trajectory is not smooth and tortuous. It is observed that the agreement is reasonable up to the stage where the simulation becomes unstable. Carrying out the numerical simulation by considering a finer mesh of 2.5 mm crack elements does not improve the results significantly.

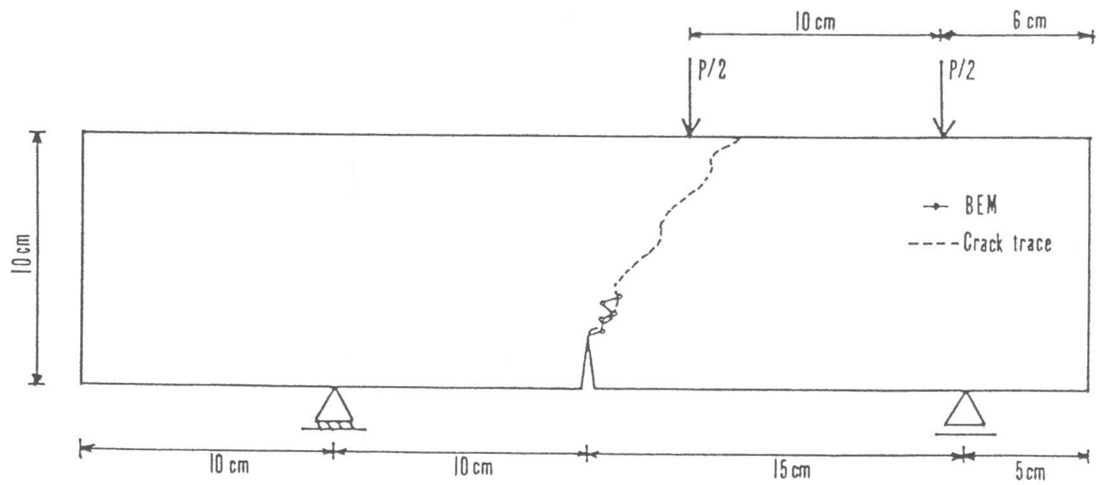

Fig. 5 Crack trajectory (center notched beam–2cm notch)

Figs. 6 and 7 show analytical results of crack trajectories obtained by the two–domain BEM in the pull–out test of anchor bolt. The analysis is carried out in 2–D. The model shown in Fig. 6 (MODEL 1) corresponds to the upper third part of the right–half portion of the concrete block. Crack propagates downwards starting from point C, along the broken line shown in Fig. 6. The downward propagation might be attributed to the short embedment length of the bolt and the relatively short distance from the point C to the bottom boundary of MODEL 1. The model in Fig. 7 (MODEL 2) represents the right–half portion of a concrete block of 90cm depth. Here, the crack propagates upwards starting from the point C and follows the solid line in Fig. 7. Crack goes upwards because of the relatively long embedment length of the bolt, the relatively short distance between the bolt and the reacting force, and proceeds far from the bottom boundary of MODEL 2. These results imply that the effect of the boundary conditions in the considered analytical model is closely related with the formation of cracks. The size of the concrete block and the set–up configuration also need close attention.

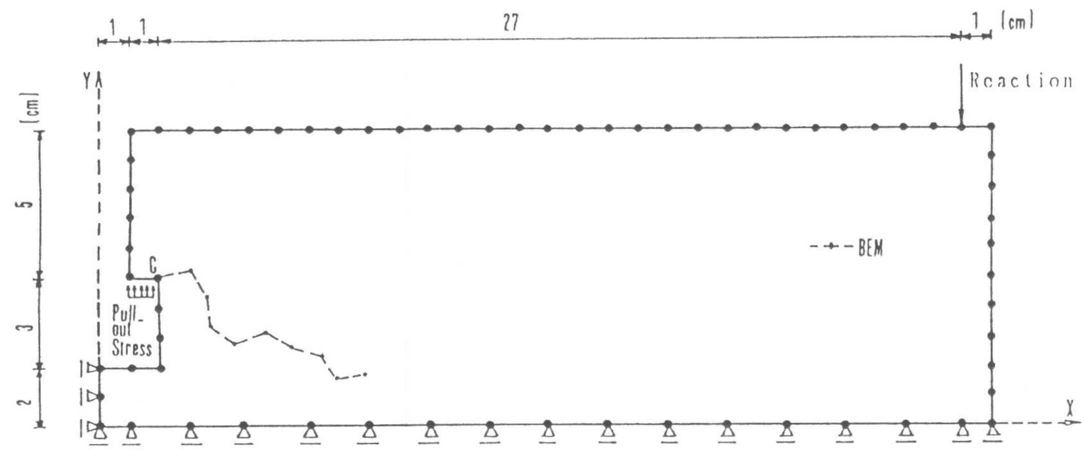

Fig. 6 Crack trajectory (anchor bolt pull-out test/ MODEL 1)

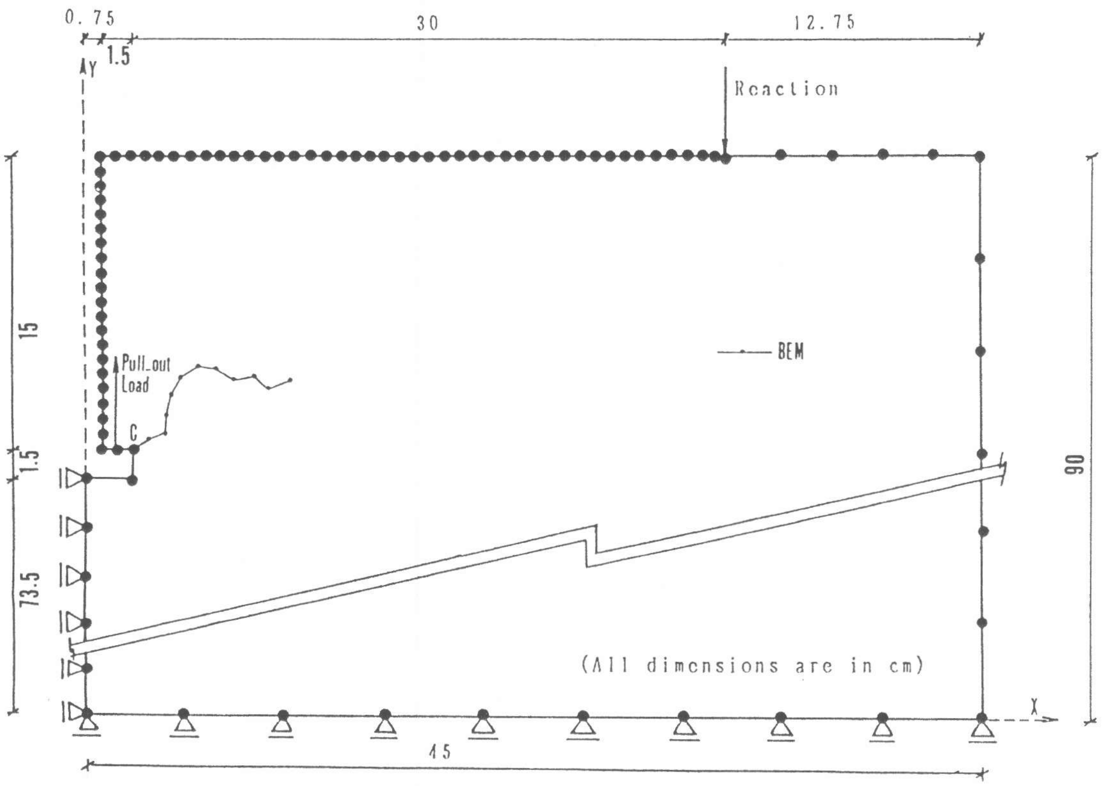

Fig. 7 Crack trajectory (anchor bolt pull-out test/ MODEL 2)

As for the load–displacement curves, the analytical results are not in good agreement with the experimental results. The load is computed as a value, which corresponds to the minimum value of load required for crack propagation. The post–peak load in all problems showed a tendency to alternately increase and decrease in value, depending on whether the direction of crack propagation is anticlockwise or clockwise. Further research is necessary for simulating a reasonable load–displacement curve of mixed–mode fracture.

4. CONCLUDING REMARKS

The applicability of multi–domain BEM analysis to predict crack trajectories for mixed–mode fracture of concrete was investigated on the basis of linear elastic fracture mechanics (LEFM). The results obtained for the cases of center–notched beams under anti–symmetric loading and pull–out test of anchor bolt embedded in a concrete block are quite suggestive. Still, further research is needed to improve the procedure, and also to clarify the size effect concerning crack propagation in pull–out tests in relation to specimen sizes and configurations.

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