#### 論 文

# [2137] MODELING OF THE QUASI-STATIC BEHAVIOR OF FRESH CONCRETE

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#### 1. INTRODUCTION

Various fresh concrete behavior can be considered as quasi-static, dewatering under pressure, bleeding and plastic shrinkage, for example. Deformability loss due to loss of water and porous surface due to water flow through the surface occur if the concrete dewaters greatly. Bleeding, causing non-uniformity, affects strength and durability of the concrete. Plastic shrinkage causes severe cracking above the fixed objects in the fresh concrete if the concrete settlement is significant.

This paper introduces an analytical model to simulate the quasi-static behavior of fresh concrete. 3-D model was developed [1], however, only the simplified 1-D model is described. The model can be used as a tool to design good fresh concrete concerning quasi-static behavior.

### 2. MULTI-PHASE MODEL AND THE RELATED GOVERNING EQUATIONS

Fresh concrete consists of various particles with different sizes and properties. In this model, fresh concrete was treated as a 5-phase material namely gravel, sand, powder, water and air phases. By assuming that the relative displacements (segregations) among the solid particles (gravel, sand and powders) are negligible, the original 5-phase material model is reduced to a 3-phase material model in which fresh concrete is considered to be composed of solid (gravel, sand and powder), liquid (water) and air phase which exists in bubble form. Though the 3-D model had been derived, the 1-D model was described here. At first, explanation of each phase is given in the following section.

#### 2.1 Solid phase

Solid phase consists of gravel, sand and powder materials. The water which always inhibits in the solid particles is considered as water as solid phase since it travels with the solid particle it inhibits. Derivation of water as solid phase will be given in the following section.

## 2.2 Liquid phase

Liquid phase is water which is regarded as being composed of 2 main

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components namely water as solid phase and water as liquid phase.

2.2.1 Water as solid phase (Vws).

Water as solid phase consists of water consumed in hydration process and water absorbed in the solid particles. These components of water are always contained in the solid phase and cannot be removed from the solid phase without applying evaporation process.

(1) Water consumed by hydration (Vwhyd). The amount of water consumed in hydration process is computed by assuming that it varies linearly with the accumulative hydration heat as ( Q is the accumulative heat of hydration at any moment and Qe is the total heat of hydration at the end of hydration process. The constant 0.25 • Wc came from the fact that the amount of water needed to complete hydration process (when Q=Qe) is about 25% of the cement weight, where Wc is weight of cement.)

$$Vwhyd = 0.25 \cdot Wc \cdot Q/Qe$$
 (1)

(2) Absorption. Absorption is defined here as the amount of water which is retained inside the solid particles and can not be removed without being forced to evaporate. Absorption was only considered for sand and gravel and is neglected for powders since it is far less than the amount of water retained at the powder surface, trapped in the flocculated structure and retained by surface tension. The amount of water absorbed by sand and gravel can be calculated using coefficients  $\beta$ f and  $\beta$ g obtained from the centrifugal water retainability test [2]. The coefficients  $\beta$ f and  $\beta$ g are the water retainability of sand and gravel at very high centrifugal level where there is no considerable change (reduction) of the coefficients even the applied centrifugal force to expel the water is increased. The amount of absorbed water, Vwab, can then be computed as (note that Wf and Wg are weight of sand and gravel, respectively)

$$Vwab = \beta f \cdot Wf + \beta g \cdot Wg$$
 (2)

2.2.2 Water as liquid phase (Vwl).

Water as liquid phase is the water component other than the water as solid phase and is composed of water of which the amount depends on liquid pressure gradient (water trapped by surface irregularity of solid, trapped in the flocculated structure and retained by surface tension) and free water.

The equation for obtaining amount of water which changes with liquid pressure gradient (Vwlp) can be written as [1]

$$Vwlp = Vwlpp(\frac{\partial p}{\partial x}, t) + Vwlpf(\frac{\partial p}{\partial x})$$
 (3)

where Vwlpp and Vwlpf represent the amounts of water which change with liquid pressure gradient for powder materials and sand, respectively,  $\partial p/\partial x$  is the liquid pressure gradient. Vwlpp is function of time since the behavior of powder materials is time dependent.

The amount of free water (Vwfr) which is movable and important for determining permeability of fresh concrete mixture can then be obtained, when the total amount of water in the mixture (Vwt) is known, from

$$Vwfr = Vwt - Vws - Vwlp$$
 (4)

#### 2.3 Air phase

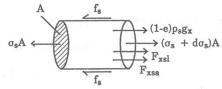
Air in fresh concrete exists in bubble form, pressure in air bubbles is in balance with the pore liquid pressure. Air volume (Va) in the mixture varies with pore pressure (u) and temperature (T) following natural law of gas as (those with subscript i are the initial values)

$$\frac{ui \cdot Vai}{Ti} = \frac{u \cdot Va}{T}$$
 (5)

### 3. ANALYTICAL PROCESS

#### 3.1 Equilibrium equations

From Fig.1, 1-D equilibrium equation of solid phase was derived as



$$\frac{d\sigma s}{dx} - \frac{dfs}{dx} + Fsl+(1-e) \cdot \rho s \cdot g = 0 \quad (6)$$

where  $\sigma$ s is stress of solid phase, fs is frictional force between solid and boundary,  $\rho$ s is density of solid phase, Fsl is drag force acting on solid by liquid, e is void ratio of the solid phase and g is gravity.

Similarly, equilibrium equation of liquid phase can be derived as

$$\frac{d\sigma l}{dx} - \frac{dfl}{dx} + Fls + S \cdot e \cdot \rho l \cdot g = 0$$
 (7)

where  $\sigma$  l is stress of the liquid phase, fl is frictional force between liquid and boundary,  $\rho$  l is density of liquid phase and Fls is drag force acting on liquid phase by solid phase. The definition of void ratio of solid phase, e, and saturation degree, S, are given as follows ( V is mixture volume, Va and Vl are volume of air and liquid in the mixture)

$$e = \frac{Va+Vl}{V}$$
 and  $S = \frac{Vl}{Vl+Va}$  (8)

Considering that boundary friction of liquid phase is negligible since it is small when compared to that of solid phase, one can combine the equilibrium equations of both phases to give the following combined equation

$$\left(\frac{d\sigma s}{dx} + \frac{d\sigma l}{dx}\right) - \frac{dfs}{dx} + (1-e) \cdot \rho s \cdot g + S \cdot e \cdot \rho l \cdot g = 0$$
 (9)

# 3.2 Constitutive equation for solid phase and boundary friction

Since liquid phase is assumed incompressible and air phase is considered to behave according to the natural law of gas, only the constitutive equation of solid phase is to be determined. Stress of solid phase is a function of strain of the solid phase. However, the stress of solid phase can be treated as a function of void ratio of the solid phase in 1-D case since strain relates linearly with volume decrease and void ratio is function of volume decrease [3,4]. Then the 1-D constitutive equation of solid phase is

$$\sigma s = \sigma s(e) \tag{10}$$

Friction between boundary and solid phase can be determined from the following equation

$$f(x) = \int_{0}^{x} \mu \cdot Ko \cdot \sigma s(x) \cdot \Phi \cdot dx/A$$
 (11)

 $\Phi$  and A are circumferential length and cross sectional area of the control unit, respectively,  $\mu$  is frictional coefficient between the boundary surface and the solid phase and Ko is the lateral pressure coefficient which can be determined from the constitutive model of solid phase.

#### 3.3 Solid-Liquid interaction

Solid-liquid interaction varies with solid-liquid relative velocity

$$Fl-s = K \cdot vls$$
 (12)

where K is the coefficient of interaction and can be nonlinear with the relative velocity between the two phases (vls). The coefficient K was proved to be related with the permeability coefficient [1]. Therefore, permeability test can be conducted to obtain  $\sigma_{\rm st}$  coefficients K.

#### 3.4 Mass balance condition

The amount of flow through a cross section, A, can be derived under mass balance condition which yields the following equation.

$$q = A \cdot vls \tag{13}$$

Solving the derived system of equations, the quasi-static behavior of fresh concrete can be simulated. Here, the constitutive equation of solid phase and the permeability relationships which represents the solid-liquid interaction are qualitatively assumed as shown in Fig. 2 and Fig. 3.

# 4. VERIFICATION OF THE QUASI-STATIC MODEL

#### 4.1 Simulation of dewatering under pressure

Test was performed by filling a cylindrical dewatering unit with fresh concrete and applying constant pressure through a piston. Dewatering was allowed at the bottom. Fig. 4 illustrates the apparatus used in the test. The testing conditions are given in Table 1.

Dewatering analysis of the tested mix-

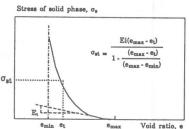


Fig.2 Assumed stress-void relation of solid phase

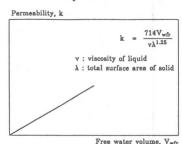


Fig.3 Assumed permeability curve

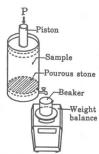


Fig.4 Dewatering apparatus

ture was conducted using the assumed solid constitutive equation and permeability equation. Comparison between analytical and test results are given in Fig.5. It can be seen that the model can qualitatively predict the dewatering of the tested mixtures of mortar and concrete.

#### 4.2. Application of the 1-D model to simulate bleeding

Table.1 Dewatering test conditions

The water migration in fresh concrete subjected to high mechanical compression had been simulated. Here, we extend our attention to bleeding which is movement of water under gravity effect as the load applied to the fresh concrete is its self weight. This can be

Mixture	Gravel	Sand	Slag	Cement	Fly ash	Pressure
	(kg/m³)					(kg/cm²)
M1	-	1260	697*	-	-	5
M2	898	725	478*	-	-	2
МЗ	898	669	410**	-	_	8
M4+	865	762	179***	180	240	5

- \* Slag with specific surface area of 3700 cm²/g
  \*\*\* Slag with specific surface area of 7700 cm²/g
- \*\*\* Slag with specific surface area of 5000 cm²/g
- \* Superplasticizer 1% of powder materials

considered as an extreme case from the view point of driving force to move water in concrete. This section introduces the application of the model to solve the bleeding problem. It should be noted that without the stress-strain model of solid phase, bleeding capacity cannot be predicted since bleeding does not stop unless total stress is sustained by the solid phase.

Fig. 6 shows the relationship between bleeding rate and w/c of cement paste with various specific surface areas of cement. The results of bleeding capacity of cement paste with varied w/c are shown in Fig. 7. The test results of cement paste were obtained from T.C. Powers [6]. illustrates the bleeding curves of concrete slab and column with very different bleeding height (slab with thickness=18cm., w/c=60% and s/a=45.5%and column with height=3m., w/c=50% and s/a=38%) and the w/c distribution

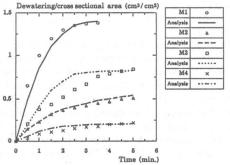


Fig.5 Dewatering results

along thickness of slab and height of column are given in Fig.9. The test results of concrete bleeding were obtained from Kanda and Yoshida [6,7]. It can be seen that the proposed quasi-static model can predict qualitatively both the bleeding rate and the bleeding capacity of the tested cement paste and concrete mixtures.

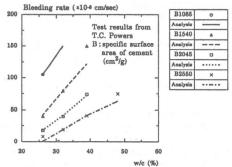


Fig.6 Bleeding rate vs. w/c of cement paste

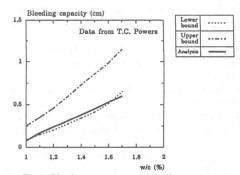


Fig.7 Bleeding capacity vs. w/c of cement paste

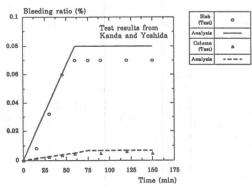
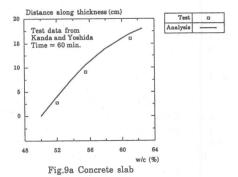


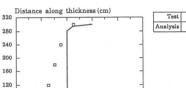
Fig.8 Test and analytical results of slab

#### 5. CONCLUSIONS

The following conclusions can be made based on the proposed quasistatic model.

- 1) A mathematical model for simulating quasi-static behavior of concrete was proposed based on multi-phase material theory.
- 2) Fresh concrete was considered to be composed of 5 phases namely gravel, sand, powder, liquid and air phases, and it was simplified to be a 3-phase





240 200 120 80 40 Test data from Kanda and Yoshida Time = 60 min. w/c (%)

Fig.9b Concrete column Fig.9 w/c distribution of concrete slab

- material model by neglecting the segregation among the solid particles.
- 3) The governing equations were derived based on the 3-phase material.
- 4) The model was used to simulate dewatering under pressure and bleeding of fresh concrete by assuming the stress-strain curve of solid phase and the permeability curve. Satisfactory qualitative results were obtained.

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