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論　文

[2128]　応力及び温度履歴に依存するコンクリートの引張強度モデル

TEMPERATURE AND TENSILE STRESS PATH DEPENDENT MODELS FOR TENSILE STRENGTH OF CONCRETE

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1.INTRODUCTION.

In the design of massive concrete structures nowadays, the precaution which gains much attention in designing procedures is the prevention of thermal crack initiation in concrete during the early period of construction. Cracking criterion of early age concrete, which is one of the important concerns in solving thermal cracking problems, was investigated in this study. In young concrete, hydration and sustained tensile stress history are two main factors affecting tensile strength. Hydration effect and sustained tensile stress history effect were separately clarified and mathematically modeled. Then, as the final step, the combined effects of the both were studied. Since temperature (directly affecting hydration) and sustained tensile stress in a concrete element in a structure keep varying in inconsistent manners, the models must be formulated by time i.e. having path dependent characteristics.

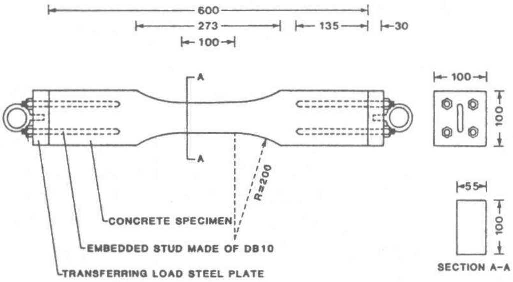

Fig.1 DIRECT TENSION SPECIMEN　UNIT (mm)

2.EXPERIMENTS.

Splitting test was used to define tensile strength of concrete in the study of strength development due to hydration. Standard cylinders(10 x 20 cm) were adopted for the test. All splitting specimens were always kept in 100% relative humidity condition all along the curing period and during loading.

For the purpose of applying sustained tensile stress while hydration was under way in the study

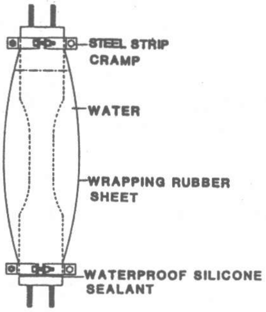

Fig.2 PREVENTION OF DRYING SHRINKAGE

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of combination of hydration and sustained tensile stress effects, direct tension test was utilized. Direct tension specimens of embedded-stud type (Fig. 1) were employed in the test. To prevent drying shrinkage, direct tension specimens tested at 20° c condition were coated with paraffin and those tested at 60° c condition were submerged in water utilizing the arrangement shown in Fig.2. Loading system used to apply sustained tensile stress is shown in Fig.3. The system is a sort of pulleys-cable system employing gravity load as source load.

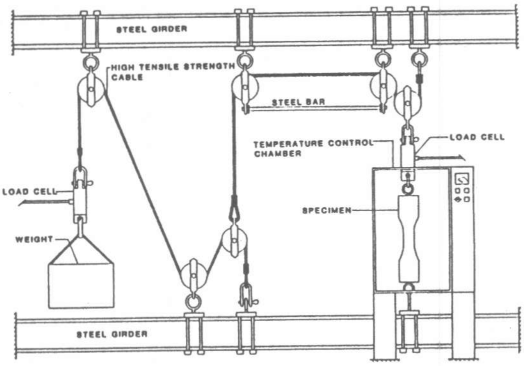

Fig.3 SUSTAINED TENSILE STRESS LOADING SYSTEM

3.GENERAL CONCEPTUAL MODEL.

Following are the basic assumptions for the conceptual model.

1. Concrete is composed of microscopic constituent elements aligned in parallel with the direction of tensile stress.

2. Number of elements in unit cross section of concrete is implied by "strength development ratio, N" defined as the ratio of tensile strength within strength development period ,the period in which the concrete strength is still developing, to the tensile strength when hydration is supposed to complete (28-day strength is assumed as the tensile strength at this state).

3. Tensile strength of early age concrete depends on two parameters.

$$Ft = N \cdot Fi \qquad (1)$$

where, Ft = tensile strength of concrete during development period
N = strength development ratio
Fi = standard tensile strength of concrete when hydration completes (supposed to be 28-day strength).

4. Degree of severeness of applied tensile stress to young concrete is defined by

$$Si = S/N \qquad (2)$$

in which, S = applied tensile stress
Si= normalized stress acting on hydrated compounds of concrete.

5. Hydration reaction increases strength development ratio while sustained tensile stress decreases it.

6. Concrete does not crack as long as the normalized stress Si is less than ultimate strength Fi and crack initiates when Si reaches Fi; i.e.,

if $Si < Fi$ no crack.
if $Si = 1$ crack occurs. ---(3)

This is the cracking criterion of early age concrete.

4.STRENGTH DEVELOPMENT MODELING.

4.1 MODEL DEVELOPMENT.

The effects of the foregoing two parameters in Eq.1 on tensile strength were studied according to the following hypotheses.

1.Strength development ratio is not affected by the changing of temperature within the range of 20° c - 60° c.

2.The tensile strength Fi is independent on temperature in 20° c - 60° c range.

3.Increasing rate of strength development ratio due to hydration (Fh) is a function of temperature and strength development ratio itself.

The first and second hypotheses were already proved by justifying experiments[6] and may be too lengthy to be mentioned here. The idea of the third hypothesis was derived from the work done by UCHIDA[2] stating that the increasing rate of accumulated heat of hydration is the function of temperature and accumulated heat of hydration at that point of time. Since hydration is the sole reaction providing strength to concrete, heat of hydration can be regarded as an indicator of the development of concrete strength and the third hypothesis was proposed as

$$Fh = dN/dt = g(T,N) = R\cdot Q\cdot N^{(R-1)/R}\cdot(1-N^{1/R}) \quad \text{------------------(4)}$$

where, Q : a material constant which varies with curing temperature
t : time duration counted from concrete casting
R : a material constant.

This hypothesis may be explained diagrammatically in Fig.4. The two dotted lines designate lines of strength development for constant curing temperatures T_1 and T_2. T_1 is higher than T_2 as being evident from the faster rate of strength development. Starting from zero time and zero N, concrete being cured at constant temperature T_2 will follow the strength development curve of temperature T_2. Then, at time t_o curing temperature suddenly changes from T_2 to T_1. According to Eq. 4, strength development rate will change from S_o of T_2 to S_1 of T_1 at strength development level of N_o. Again, at time t_1, curing temperature abruptly changes from T_1 to T_2 and strength development rate switches from S_2 of T_1 to S_3 of T_2. Consequently, if the strength development curves for any constant curing temperature could be defined, the development curve for a given arbitrary temperature history would be able to be traced based on this hypothesis. Integrating Eq.4 by regarding that Q is constant with time (constant curing temperature), a function for strength development under any constant curing temperature can be obtained as

$$N = (1-e^{(-Qt)})^{R} \quad \text{----------------------(5)}$$

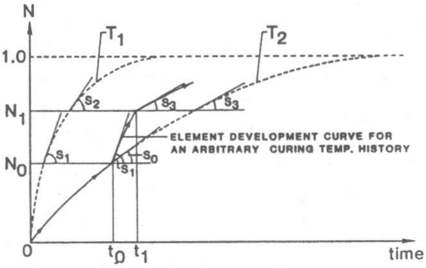

Fig.4 THE THIRD HYPOTHESIS

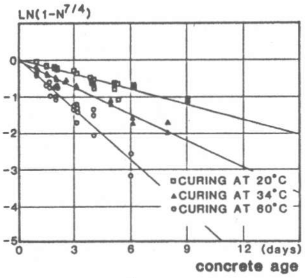

Fig.5 REGRESSION ANALYSIS RESULT

An experiment was conducted to determine in what manner Q represented the effect of curing temperature on strength development. Three sets of tensile strength development data corresponding to 20° c, 34° c and 60° c curing temperatures were obtained from the experiment. Regression analysis was made to fit the obtained data with Eq.(5) in Fig.5 in which R was chosen to be 4/7 for the best correlation and to satisfy the zero strength condition at casting time.

Experimental results shown in Fig.5 yielded three "Q"s corresponding to three curing temperatures. The pairs were plotted in Fig.6 and, coin-

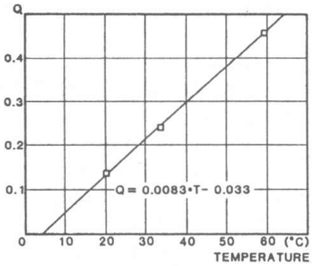

Fig.6 Q - TEMPERATURE RELATION

cidentally, the relation came out to be a sort of straight line relation as

$$Q = 0.0083 \cdot T - 0.033 \quad \text{--------------------(6)}$$

Thus, development curves for any constant curing temperature can be defined using this Q-temperature relationship. Now that Q-temperature relationship (Eq.6) and rate of strength development (Eq.4) were developed, the modeling would be completed by employing an appropriate step by step numerical integration.

An experiment was carried out to verify the model. Many cylindrical specimens cured under various arbitrary temperature histories were tested for splitting strength and the strengths were compared with the predicted ones from the model in Fig.7.

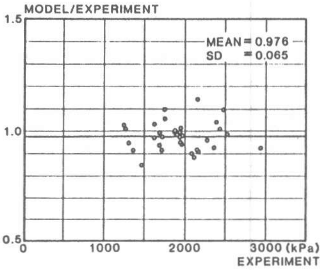

Fig.7 MODEL VERIFICATION

4.2 COMPARISON WITH THE MATURITY CONCEPT.[5]

Figure 8 shows the relationship between tensile strength Ft and maturity M obtained by conducting regression analysis. Figure 9 is the result when the concept was applied to predict tensile strengths of concrete. There are two groups of data in the figure. One is the group of splitting strengths of specimens cured under constant temperatures and the other is that of specimens cured under varying temperature histories.

Also shown in Fig.10 is the comparative result using the proposed model to predict the strengths of the same two groups of data. It can be obviously seen that both maturity and the model could be applied very well to the cases of concrete cured under constant temperatures. However, the maturity concept fell short of good prediction in the cases of fluctuating curing temperature histories while the proposed model could still be well applied.

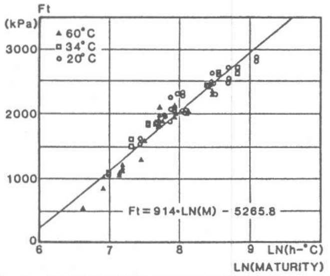

Fig.8 STRENGTH MATURITY RELATION

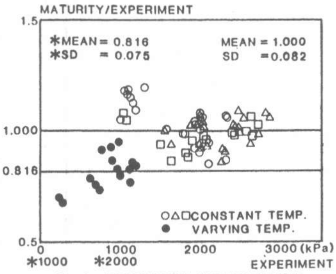

Fig.9 MATURTY PREDICTION

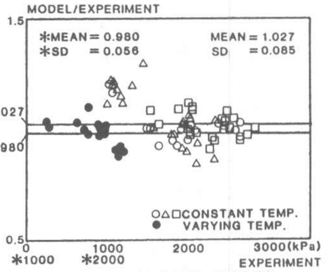

Fig.10 PROPOSED MODEL PREDICTION

5. MODELING OF SUSTAINED TENSILE STRESS EFFECT.

Shown in Fig.11 is the result of experiment done by FOURE[4] who had done an experiment on direct tension specimens at ages of 7, 14, 28 and 91 days. The experiment was done in atmospheric condition (temperature 15° c-20° c, relative humidity 55%) and there was no supply of water to the concrete specimens. Each specimen in test was loaded at a constant sustained tensile stress(σ_u) below tensile strength(Fi) until

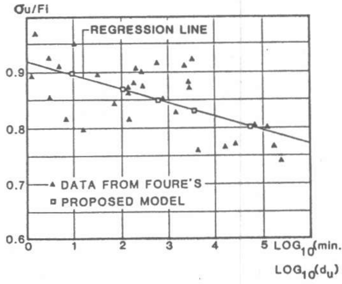

Fig.11 SUSTAINED STRENGTH DATA

failure. Many specimens were tested varying level of applied sustained stresses. Data presented in Fig.11 are plotted points between σ_u/Fi and logarithm of d_u, duration between application of the sustained stress and failure.

In order to represent the reduction of strength due to sustained stress history, it was assumed that the strength development ratio N decreases due to the accumulated damage in concrete caused by the sustained stress and, thus, strength development rate Fs (negative) due to sustained stress was assumed as a function of stress intensity.

$$Fs = dN/dt = A \cdot \tan(\pi/2 \cdot \alpha^{B}) \quad \text{----------------------------------}(7)$$

where, α = Si/Fi : stress intensity
A,B : material constants.

Using Eq.7 for the negative rate of strength development ratio in a step by step numerical integration, a model simulating breaking process could be realized. The model was verified using FOURE's data by analyzing five representative cases of specimens supposed to be subjected to 0.9Fi, 0.87Fi, 0.85Fi, 0.82Fi and 0.80Fi of constant sustained stresses with the values of A,B material constants as 3.4 and 76 respectively. The outcome of the analysis is included in Fig.11.

6.LINEAR COMBINATION OF HYDRATION AND SUSTAINED TENSILE STRESS EFFECTS.

The final expected goal of this study was to combine the creative effect of hydration and the destructive effect of sustained tensile stress. In order to model the physical behavior of young concrete subjected to sustained tensile stress, both explained models must be combined. Since there might be confusion of the strength development ratio N, those in Eq.2, 4 and 7 were newly defined respectively as

N : Net strength development ratio
dNh: Increasing strength ratio from hydration
dNs: Decreasing strength ratio due to sustained tensile stress.

The important point is how to combine dNh and dNs to obtain the change of net strength ratio dN in a time interval dt. As the first trial, simple linear combination of the two was chosen. Thus,

$$dN = dNh - dNs \quad \text{--------------------------}(8)$$

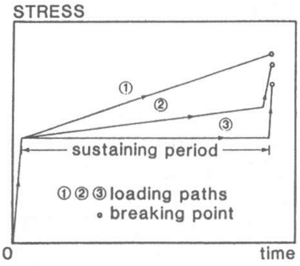

Fig.12 TYPICAL STRESS PATHS

Following this combination scheme and using an appropriate step by step numerical integration ,the model of the combined effects could be achieved.

An experiment was also carried out to study the combination effects on concrete of ages 3-9 days. Testing procedures were that a direct tension specimen would be loaded at about 80% of starting strength and then the load would be sustained or gradually increased until failure (this tensile strength will be referred to as sustained strength). The typical applied stress paths are shown in Fig.12. At failure, concrete cylinders which were kept in exactly the same curing condition as the failed direct tension specimen but in stress-free state would be

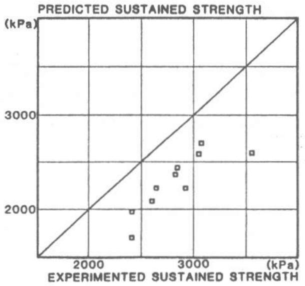

Fig.13 PREDICTION RESULT

tested for splitting strength using standard rate of loading (this strength will be called standard strength) to be compared with the sustained strength. The tensile stress sustaining period ranged from half a day to about five days and the specimens were tested in both 20° c and 60° c environments.

Prediction result of the proposed combination model is shown in Fig.13. It can be seen from the figure that the predicted strengths are lower than the experimented ones. One interesting point is that the standard strength is lower than the sustained strength of concrete cured in the same condition as can be seen by comparing both strengths as shown in Fig.14. This might be the reason why the proposed model using the simple linear combination (Eq.8) failed to predict the combined effects. The model did not take into account the recovery of broken elements and non-linear influence of sustained tensile stress on hydration which appeared to be a "hardening" effect.

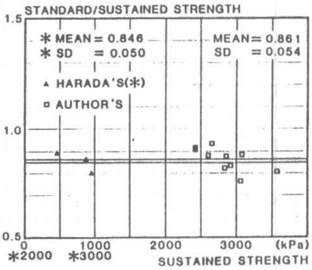

Fig.14 COMPARISON OF STRENGTH

7.CONCLUSION.

Curing temperature path dependent model which can correctly predict tensile strength development in early age concrete subjected to an arbitrary curing temperature history was proposed. Also, tensile stress path dependent model to predict breaking point of concrete with no hydration subjected to sustained tensile stress was proposed and verified. The trend that sustained strength is higher than standard strength indicates that sustained tensile stress may have somehow positive effects instead of absolutely negative effects on strength of young concrete.

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