論 文
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[2127] 任意形断面を有するPRC部材のひびわれ強度と変形

CRACKING STRENGTH AND DEFORMATION OF PARTIALLY PRESTRESSED CONCRETE MEMBERS WITH ARBITRARY CROSS SECTIONS

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1. INTRODUCTION

Recently the use of partially prestressed concrete has gradually increased. Partial prestressing offers better control of cracking, deflection and camber, and higher ductility, energy absorption and cost saving than fully reinforced concrete. In order to perform rational design of partially prestressed concrete members, simple and systematic stress analysis has been demanded.

In this paper, cracking strength and deformation as the important structural index of limit states of a partially prestressed concrete member with arbitrary concrete section considering creep and shrinkage of concrete and relaxation of prestressing steel were analysed. The procedure of the analysis was demonstrated by numerical example for multi-hollow slab bridge.

2. ANALYSIS

In order to obtain the cracking strength and deformation of partially prestressed concrete considering prestress losses, structural behavior of partially prestressed concrete as shown in Fig. 1 was devided to the following five load stages;
First stage : Just after applying sustainded load due to dead load and initial prestress, Second stage: Changes of stress, strain and curvature due to prestress losses, Third stage: Decompression, Fourth stage: Just before cracking, Fifth stage: Changes of strain and curvature after cracking. It is assumed that a) plane cross section remains plane after loading, b) Hooke's low is applicable every stage, c) concrete does not resist tensile stress after cracking, and d) modulus of elasticity of prestressing steel equal to non-prestressing steel.

2.1 STEPS OF ANALYSIS

1) First stage: Just after applying sustainded load due to dead load and initial prestress

Equations for the axial strain ε_{oi} and the curvature ϕ_i in an arbitrary concrete section subjected to a bending moment M and an axial force N at zero axis as shown in Fig. 1 a) are given

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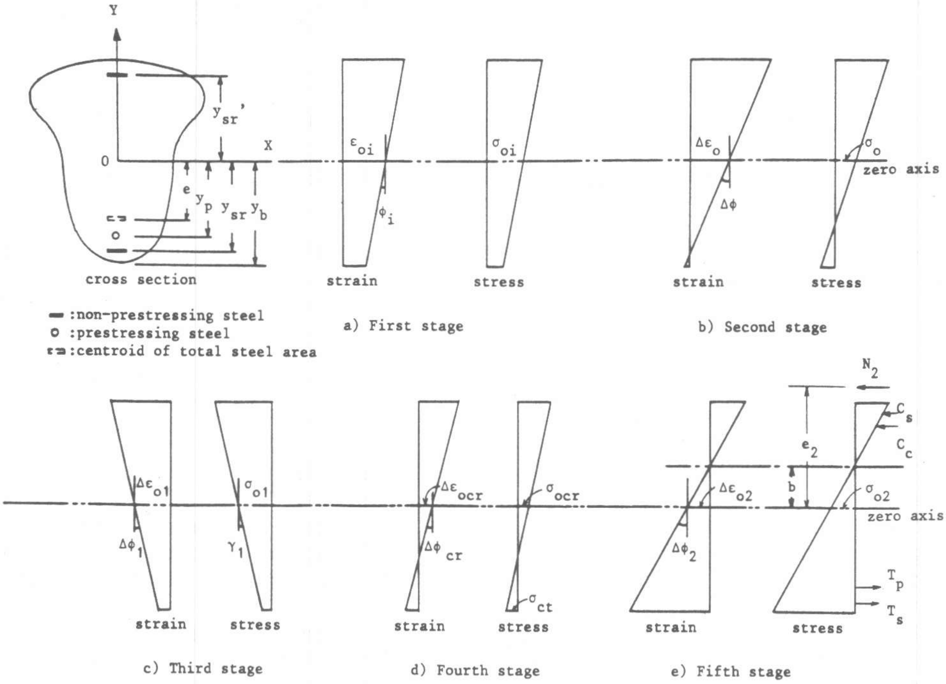

Fig. 1 Five Load Stages of Partially Prestressed Concrete

$$\int \sigma dA = E_c \varepsilon_{oi} \int dA + E_c \phi_i \int y dA = N \quad (1)$$

$$\int \sigma y dA = E_c \varepsilon_{oi} \int y dA + E_c \phi_i \int y^2 dA = M \quad (2)$$

Eqs. (1) and (2) can be rewritten as follows:

$$\begin{Bmatrix} \varepsilon_{oi} \\ \phi_i \end{Bmatrix} = \frac{1}{E_c(AI-G^2)} \begin{bmatrix} I & -G \\ -G & A \end{bmatrix} \begin{Bmatrix} N \\ M \end{Bmatrix} \quad (3)$$

in which ε_{oi} is strain, ϕ_i is curvature, A is transformed area, G and I are its first and second moment about a zero axis and E_c is elastic modulus of concrete, respectively.

2) Second stage: Changes of stress, strain and curvature due to prestress losses

The changes of stress of concrete, prestressing steel, non-prestressing steel due to creep and shrinkage of concrete and relaxation of prestressing steel were considered. The sum of the force increment in the above three materials must be zero.

$$\Delta P_c = -(\Delta P_p + \Delta P_s) \quad (4)$$

in which, Δ is increment, P is axial force and subscript c, p and s are concrete, prestressing and non-prestressing steel, respectively. The changes of strain in the concrete, prestressing and non-prestressing steel at the centroid of total steel area must be compatible, thus,

$$\varepsilon_{sh} + \frac{\psi\sigma_{cp}}{E_c} + \frac{(1+\rho\psi)}{E_c}\frac{\Delta P_c}{A_c}\left(1+\frac{e^2}{r^2}\right) = \frac{(\Delta\sigma_p - \Delta\sigma_{pr})}{E_s} = \frac{\Delta\sigma_s}{E_s} \qquad (5)$$

in which ψ is creep coefficient, $\Delta\sigma_p$ and $\Delta\sigma_s$ are stress increments in prestressing and non-prestressing steel, $\Delta\sigma_{pr}$ is relaxation, σ_{cp} is stress due to sustainded load, ρ is aging coefficient depending on the age of concrete[1,2], ε_{sh} is shrinkage and $r^2=I_c/A_c$.

From Eqs. (4) and (5), ΔP_c is given as follows:

$$\Delta P_c = -\frac{n\psi\sigma_{cp}A_{st} + E_s\varepsilon_{sh}A_{st} + \Delta\sigma_{pr}A_P}{1+\alpha\frac{nA_{st}}{A_c}(1+\rho\psi)}$$

in which $n=E_s/E_c$, $A_{st}=A_p+A_s$, A_p and A_s are area of prestressing and non-prestressing steel.

Changes of strain $\Delta\varepsilon_o$, stress σ_o and curvature $\Delta\phi$ due to prestress losses are

$$\Delta\varepsilon_o = \varepsilon_{sh} + \varepsilon_{oi}\psi + \frac{\Delta P_c}{A_cE_c}(1+\rho\psi) \qquad \Delta\phi = \phi_i\psi + \frac{\Delta P_c e}{E_cI_c}(1+\rho\psi)$$

$$\sigma_o = \Delta P_c/A_c$$

and the change of stress $\Delta\sigma_p$ in a layer of prestressing steel is

$$\Delta\sigma_p = E_s(\Delta\varepsilon_o + y_p\Delta\phi) + \Delta\sigma_{pr}$$

3) Third stage: Decompression

As the stress of concrete for the state of decompression is given by the stress of the second stage superimposed on that of the first stage, axial force N_1 and bending moment M_1 are calculated by referring to Fig. 1 c),

$$\begin{Bmatrix} N_1 \\ M_1 \end{Bmatrix} = -\begin{bmatrix} A & G \\ G & I \end{bmatrix}\begin{Bmatrix} \sigma_{o1} \\ \gamma_1 \end{Bmatrix}$$

Changes of strain $\Delta\varepsilon_{o1}$ and curvature $\Delta\phi_1$ are written as follows:

$$\Delta\varepsilon_{o1} = -\sigma_{o1}/E_c \qquad \Delta\phi_1 = -\gamma_1/E_c$$

4) Fourth stage: Just before cracking

It is assumed that the cracks occur when the concrete stress of bottom face reaches flexural tensile strength σ_{ct}. The axial force $N_2=-N_1$ and cracking moment M_{cr2} are applied on the concrete cross section of decompression state. The concrete stress of bottom face due to axial force N_2 and cracking moment M_{cr2} is

$$\sigma_{ct} = -\frac{N_2}{A} + \frac{M_{cr2}y_b}{I}$$

Consequently, the cracking moment in this stage is

$$M_{cr2} = (\sigma_{ct} + \frac{N_2}{A}) \frac{I}{y_b}$$

Cracking moment can be obtained by superimposing moments calculated from first stage to fourth stage.

5) Fifth stage: Changes of strain and curvature due to cracking

After decompression, axial force N_2 and bending moment $M_2=M_L-M_1$ apply on the concrete cross section of decompression state, in which M_L is bending moment due to live load. The beam can be analysed as an ordinary reinforced concrete member subjected to an eccentric compression force N_2 with the distance $e_2=M_2/N_2$. These force N_2 and moment M_2 produce the following changes of strain and curvature

$$\begin{Bmatrix} \Delta\varepsilon_{o2} \\ \Delta\phi_2 \end{Bmatrix} = \frac{1}{E_c(AI-G^2)} \begin{pmatrix} I & -G \\ -G & A \end{pmatrix} \begin{Bmatrix} N_2 \\ M_2 \end{Bmatrix}$$

2.2 EQUILIBRIUM EQUATION

Equilibrium of axial force and bending moment are expressed as follows:

$$N_2 = C_c + C_s - T_s - T_p \tag{6}$$

$$M_2 = N_2 e_2 \tag{7}$$

Eliminating N_2 from two Eqs. (6) and (7) and rewriting the equation to be used for bisection method as follows:

$$f(b) = I_{xc} + nI_{xse}' + nI_{xse} + nI_{xpe} - b(G_{xc} + nG_{xse}' + nG_{xse} + nG_{xpe})$$
$$-e_2\{G_{xc} + nG_{xse}' + nG_{xse} + nG_{xpe} - b(A_c + nA_s' + nA_s + nA_p)\} \tag{8}$$

in which subscript xc is compressive concrete, xpe and xse are prestressing and non-prestressing steel, prime denotes compression. By adapting Gauss' integral to Eq. (8), the results are approximately calculated by the linear integrals on the cross section of connected straight lines as shown in Appendix[3].

2.3 DEFLECTION

The total strain and curvature are given by superposition of first, second, third and fifth stages,

$$\varepsilon_o = \varepsilon_{oi} + \Delta\varepsilon_o + \Delta\varepsilon_{o1} + \Delta\varepsilon_{o2}$$

$$\phi = \phi_i + \Delta\phi + \Delta\phi_1 + \Delta\phi_2$$

Assuming the prestressing tendon's profile in the beam is parabora, the deflection at mid-span can be represented by the following equation,

$$\delta_{max} = \frac{5}{48} \ell^2 \phi$$

in which ℓ is length of span.

3. NUMERICAL EXAMPLE

Cracking moment and deformations of multi-hollow slab bridge of span length 20 m at one year illustrated in Fig. 2 were calculated. By giving dimensions and physical characteristics of the materials used, numerical results were obtained as shown in Table 1.

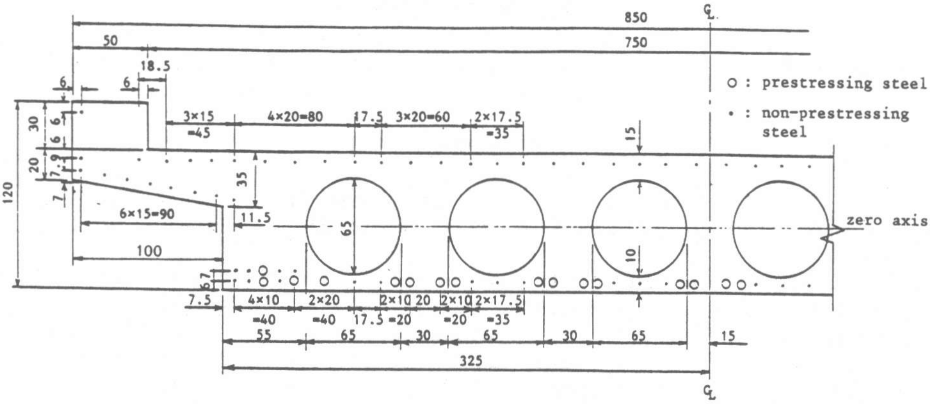

Fig. 2 Cross Section of Hollow Slab Bridge

$A_s = 94 \times 2.84 = 266.96\ cm^2$ Initial Prestress $28 \times 35.5 = 994.0$ tf

$A_p = 28 \times 6.0324 = 168.9072\ cm^2$ $\rho = 0.8$ (at one year)

$M_d = 588.63$ tf·m $\psi = 2.0$

$M_1 = 368.13$ tf·m $\varepsilon_{sh} = 200 \times 10^{-6}$

$E_c = 350000\ kgf/cm^2$ $\Delta\sigma_{pr} = 294.27\ kgf/cm^2$

$E_s = 2100000\ kgf/cm^2$ $\sigma_{ct} = 40.0\ kgf/cm^2$

Table 1 Numerical Results of Hollow Slab Bridge

Stage	Strain at zero axis	Curvature	Strain at top face	Strain at bottom face	Stress at zero axis	Stress at top face	Stress at bottom face
	10^{-6}	$10^{-6}\ cm^{-1}$	10^{-6}	10^{-6}	kgf/cm^2	kgf/cm^2	kgf/cm^2
1	-58.57	0.6968	-104.8	-21.22	-20.50	-36.69	-7.428
2	-274.0	2.056	-410.6	-163.8	5.804	-0.120	10.59
3	41.99	-0.9516	105.2	-9.020	14.69	36.81	-3.157
4	-41.21	1.364	-233.9	114.3	-14.42	-81.85	40.00
5	140.9	9.306	-477.1	639.7	49.31	-167.0	223.9

The cracking strength is 943.165 tfm and the deflection at the mid-span due to full service load is 4.628 cm.

4. CONCLUSIONS

The cracking strength and deformation considering to time dependent losses were analysed in this study. The results obtained by this study are summarized as follows:

1) Cracking strength and deflection of partially prestressed concrete with arbitrary cross section were analysed and the computer program was

developed.
2) Prestress loss at arbitrary time was easily calculated, since aging coefficient was used in this analysis.
3) Initial prestress level designed for partially prestressed concrete can be evaluated by the analytical results.

ACKNOWLEDGEMENT

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REFERENCES

[1] Bazant, Z. P., "Prediction of Concrete Creep Effects Using Age-Adjusted Effective Modulus Method", ACI Journal, Proceedings, Vol.69, No.4, 1972, pp.212-217.
[2] Amin Ghali, et al, "Partially Prestressed Concrete Structures", Journal of Structural Engineering, Vol.111, No.8, 1985, pp.1846-1865.
[3] KAWAKAMI, M., et al., "Limit State Analysis of RC and PC Member Subjected to Combined Flexure and Direct Compression", International Symposium on Fundamental Theory of Reinforced and Prestressed Concrete, Vol.1, 1986, pp.102-109.

APPENDIX

Gauss' integral

$$H_{mn} = \iint x^m y^n dA \quad \text{(A1)}$$

$$= \int \frac{x^m y^{n+1}}{n+1} dx \quad \text{(A2)}$$

Referring to Fig. 3,

$$t = x - x_i, \quad y = y_i + \frac{\Delta y_i}{\Delta x_i} t \quad \text{(A3)}$$

and substituting Eq. (A3) into (A2),

$$\Delta_i H_{mn} = \frac{1}{n+1} \int_0^{\Delta x_i} (x_i + t)^m \cdot \left(y_i + \frac{\Delta y_i}{\Delta x_i} t\right)^{n+1} dt$$

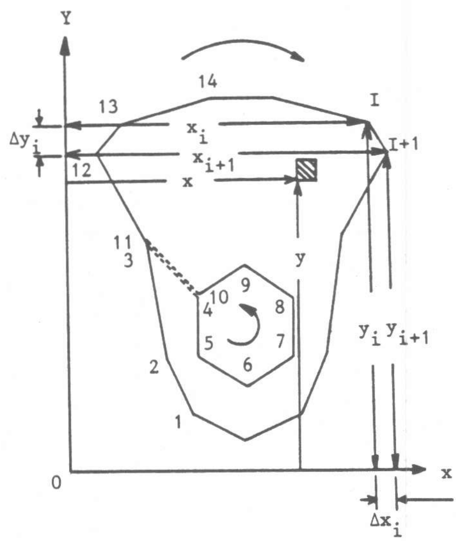

Fig. 3 Contour and Numbering of Arbitrary Concrete Section

$$A = H_{00} = \frac{1}{2} \Sigma (x_{i+1} y_i - x_i y_{i+1})$$

$$G_x \fallingdotseq H_{01} = \frac{1}{2} \Sigma (x_{i+1} - x_i)\{y_i^2 + \frac{1}{3}(y_{i+1} - y_i)(y_{i+1} + 2y_i)\}$$

$$I_x = H_{02} = \frac{1}{3} \Sigma (x_{i+1} - x_i)\{y_i^3 + \frac{1}{6}(y_{i+1} - y_i)(y_{i+1} + 2y_i)^2 + \frac{1}{12}(y_{i+1} - y_i)^3\}$$